6-2020

Derivation and Exploration of Analytical Aircraft Stability Analysis

Roderick Landreth
Union College - Schenectady, NY

Derivation and Exploration of Analytical Aircraft Stability Analysis

By

Roderick E. Landreth

Submitted in Partial fulfilment
of the requirements for
Honors in the Department of Mechanical Engineering

Union College
June 4th, 2020

Abstract

LANDRETH, RODERICK: Derivation and Exploration of Analytical Aircraft Stability Analysis. Department of Mechanical Engineering, June 2020

ADVISOR: Professor Rebecca Cortez

A comprehensive measure of the in-flight stability of an aircraft is essential, though it can take a large amount of resources to do accurately and usefully. Small companies and colleges often don't have the resources to attain accurate experimental or analytical data to describe stable flight before potential failure. This composition investigates methods of completing a static and dynamic stability analysis for an aircraft feasible for a college, small business or hobbyist to complete, with the goal to eliminate the 'barrier of entry' concerning a lack of information and resources. A sample analytical calculation for static stability of Union College's 2020 SAE Aero Advanced plane shows positive static stability, and a sensitivity analysis displays the effect of changing the craft dimensions. This plane was used because it has been flown and the analytical calculation confirms flight test observations. Dynamic stability methods are presented and evaluated for their benefits, and software that completes the same are recommended. The static analysis and sensitivity analysis is used to describe problem geometries often seen in planes and some solutions. This is then used to make educated dimensional alterations. This thesis should aid in the design process by listing an evaluating initial 'rule of thumb' relations, creating a Matlab stability algorithm, and providing the sources to research and complete analysis for dynamic stability and control.

Contents

List of Tables

List of Figures

List of Acronyms

CG = Center of Gravity
AC = Aerodynamic center
F_L = Lift Force
$F_{L\ w}$ = Lift Force of the Wing
$F_{L\ t}$ = Lift Force of the Tail
F_D = Drag Force
$F_{D\ w}$ = Drag Force of the Wing
$F_{D\ t}$ = Drag Force of the Tail
F_{th} = Force of Thrust from the plane motor
F_g = Forge of gravity on an object
C_m = Moment produced by the airfoil in-plane with the chord
$C_{m\ cg}$ = Moment produced by the airfoil in-plane with the chord at the CG
M = A general moment
$M_{ac\ w}$ = The moment in-plane with the wing chord at the AC of the wing
$M_{ac\ t}$ = The moment in-plane with the tail chord at the AC of the tail
C_L = Lift Coefficient (subscript can denote wing or tail) (Unitless)
C_D = Drag Coefficient (subscript can denote wing or tail) (Unitless)
$C_{Mc\ ac}$ = Moment coefficient at the AC of the airfoil (Unitless)
$A_t = A_{\text{top wing}}$ = Top Planform Area (Max horizontal area lift acts on), will indicate of wing or tail
A_f = Front Planform Area (Max Vertical area list acts on)
ρ = Density (if not specified, assume density of air)
v = Speed (or velocity, assumed direction forwards) of the plane relative to surrounding air
α = Angle of attack
$d\alpha$ = A small increment in angle of attack
α_{plane} = Angle of attack of the whole plane
α_{th} = Angle of attack of the thrust force vector
$\alpha_{I\ w}$ = Angle of incidence of the wing
$\alpha_{I\ t}$ = Angle of incidence of the tail
$\alpha_{I\ \text{Motor}}$ = Angle of incidence of the motor
$l_{tail} = l_t$ Moment arm of the tail from the center of gravity to the AC of the tail
xac_w = Moment arm of the ting from the center of gravity to the AC of the wing
x_{motor} = Distance from the motor to the CG b = The length (span) of the lifting surface (subscript indicates wing or tail)
c = The width (chord) of the lifting surface (subscript indicates wing or tail)
I_{xx} = Mass moment of inertia of the plane about the Roll axis
θ = Plane roll angle
$\ddot{\theta}$ = Plane roll angle second derivative with respect to time

Chapter 1

Background and Introduction

Aircraft design in industry requires not only through testing and consideration of materials and airfoils, payload and thrust calculations, configuration selection and mechanisms to move control surfaces, drop payload, release gliders, steer on the ground, etc., but these decisions need to be made with respect to the stability of the craft while in air. The aerodynamics of a plane rely heavily on its response to disturbances while in air, produced by gusts of wind or sudden changes in a pilot's control. Being a heavily developed and impactful field of research, stability considerations are not made purely by testing the craft in flight. The time and resources that could be wasted upon failure would be an insurmountable barrier for hobbyists and smaller businesses if this were the case. The response of an aircraft can be broken into measures of static and dynamic stability, and although it is easiest to test these in flight to high accuracy, analytical and computational methods are commonly used, often even simplified into rules of thumb for use by hobbyists. Aircraft design decisions have to be made with respect to the static and dynamic stability of a plane, and although the most accurate data is still resource intensive to produce, analytical analysis using plane geometry alone are feasible for small companies or students without access to graduate level courses, and are much more accurate than rules of thumb.

The purpose of this composition is to provide information on static and dynamic stability for general aviation aircrafts with a focus on small radio controlled crafts used for the SAE Aero competition. This manuscript is written for students or small businesses that don't have professional grade equipment or ways to attain graduate level knowledge in the hope to overcome this ’barrier of entry.’ In addition to theory, sample calculations for static stability and a script will follow. The script completes the same calculations, but can be applied to different geometries as an easy recourse for future students. Dynamic stability is investigated in as far as what its components are, the results, and how to display them, though a full derivation is beyond the scope of this report.

1.1 Background Aerodynamics Information

Because this discussion deals primarily with the motion of a plane, it is beneficial to understand the basics of plane flight for referencing later on. As with any system on earth, the force of gravity has an affect on a plane. To combat this, a plane generates thrust either pulling or pushing it forwards. This causes air to flow over the wings, bringing a couple more forces into the foreground of importance; the craft would then generate lift and drag. Often, fluid dynamics textbooks such as reference [1] cover this topic in detail.

Lift is the force opposing gravity, caused by pressure differences on top and bottom of the plane. While there is some lift generated because of the fuselage of a plane, the force of lift is usually approximated by equation 1. In this, ρ is the density of air, v is the speed of the plane with respect to the air, A_t is the cross sectional area through the wings parallel to the air flow, and C_L is the lift coefficient. This is a dimensionless parameter that changes with the wing shape and angle the wing meets the air stream, which is called the angle of attack (α). The drag force opposes the direction of motion, and although there are many different components caused by the wing roughness or vortices caused by the wings, the most common approximation is equation 2. Similarly, C_D denotes the coefficient of drag and A_f is the cross section area of the wing perpendicular to the air flow.

$$F_L = C_L \rho \frac{v^2}{2} A_t \tag{1}$$

$$F_D = C_D \rho \frac{v^2}{2} A_f \tag{2}$$

The lift and drag coefficients are complex values that change with the shape of the wing, the angle of the plane, weather, air density, etc., but commonly they are approximated at a given angle of attack for an airfoil. The roughly whale shape of a wing that the air flows parallel to is called an airfoil. Although a plane doesn't need an airfoil to fly, this is a highly optimized shape that causes a larger pressure difference between the top and bottom of the wing, and generally a larger lift coefficient. There are many databases that show 2-D analysis of these shapes, plotting C_L and C_D vs α, the angle of attack. Because the air rushing over the airfoil causes lift due to a pressure difference, parts of the airfoil are pushed on harder than others, and these don't often balance each other out completely. An airfoil often has some net moment, also plotted vs α, and it is the job of the tail, placement of the plane's center of gravity, and other mechanisms to negate this for stable flight.

A discussion of longitudinal stability will be mentioned later on, so it is important to discuss the various axes of a plane. The axis leading from the nose to the tail is the roll axis, rotating around this axis causes the plane to roll. The pitch axis intuitively controls the angle the nose is pitched at, and runs parallel to the wings of a conventional aircraft. The yaw axis is straight up and down through a plane. Longitudinal stability concerns the plane's motion about the roll axis, and lateral stability concerns the plane's motion about the pitch axis.

1.2 Defining Static and Dynamic Stability

The difference between static and dynamic stability is the difference between the immediate response and the response over time. If your first response is to swerve away from an oncoming obstacle in the road, one doesn't want to afterwards over-correct and hit something else. Static stability is the instantaneous response, in this case of a plane that has been disturbed from its stable flight without alterations of control surfaces. In other words, with 'stick fixed' analysis, the response of the plane relies on the centers of gravity and airfoils and physical configuration of the plane to maintain stable flight. A plane is only statically stable when it has a restoring moment directing the craft back to its equilibrium angle. This would mean at a given angle of attack, a moment is created that pushes it back to its equilibrium angle. Dynamic stability then is what happens afterwards; it characterizes the response of the craft over time, describing if and how it returns to the initial state after the disturbance. For a craft to be stable over time, it must be stable in the moment; static stability is needed for dynamic stability but alone is not enough to ensure it [2].

Both types of stability data are visually displayed differently, and have different effects on the craft in flight. A craft with desirable static and dynamic stability produce a plane that oscillates upon disturbance towards its original trajectory, and these oscillations eventually decay until the craft is flying as it was before. A craft with undesirable stability characteristics exhibits oscillations that increase in magnitude until the craft can no longer maintain flight. Stability is not binary, however. The cases with stability somewhere in between can be broken into different modes, characterizing many ways the plane responds.

1.3 Stability Modes

For both static and dynamic stability, there are unique behaviors that the craft can experience, usually classified into positive, neutral, and negative stability [2]. Figure 1 gives an example of what this would look like for a ball system affected by gravity.

For static stability, positive indicates that the craft has an initial restoring moment to return to equilibrium. As can be inferred from this, neutral indicates very little moment, and negative indicates that the craft's moment makes it diverge further from equilibrium.

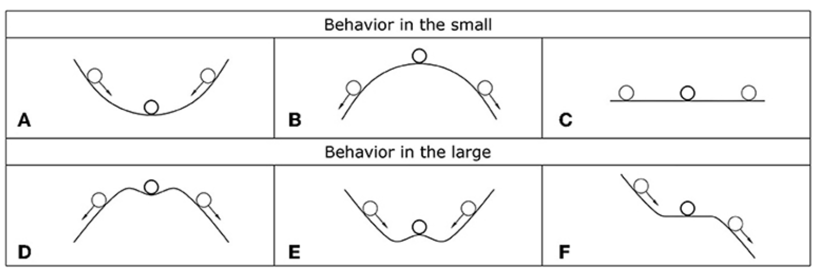


Figure 1: As a dynamic system, (A) is an example of positive static stability because the a gravitational restoring force would push it towards its initial point, (C) has very little force affecting it indicating neutral static stability, and (B)'s ball would actively accelerate away from equilibrium if disturbed, indicating negative static stability. In reality, more complex systems will could have this behavior only within a certain range of angles, presented by (D), (E), and (F) [3]. (Mascolo, I., 2019, "Recent Developments in the Dynamic Stability of Elastic Structures," Frontiers in Applied Mathematics and Statistics, 5(1), pp. 1-10.)

Dynamics stability's responses by the same names don't classify any direction of motion or moment, but instead the behavior of the oscillations over time. In system dynamics, a damped system is one that has a component of force that opposes motion, like a spring moving a block that is slowed down by friction. This system is oscillatory, with positive dynamic stability exponentially decaying towards equilibrium (Overdamped). As can be inferred, neutral dynamic stability indicates something called the 'damping ratio' is small and there is little change in the amplitude of the oscillations (Undamped), and negative indicates a negative 'damping ratio,' so that the amplitude increases out of control (Negatively damped). The damping ratio is a common term for a parameter in second order differential equations (because these are moments and forces, which can be broken down into mass and acceleration) which governs how fast motion decays [4]. It's use is explained more in section 1.6 and 3.2. Modes of dynamic stability are visualized in figure 2.

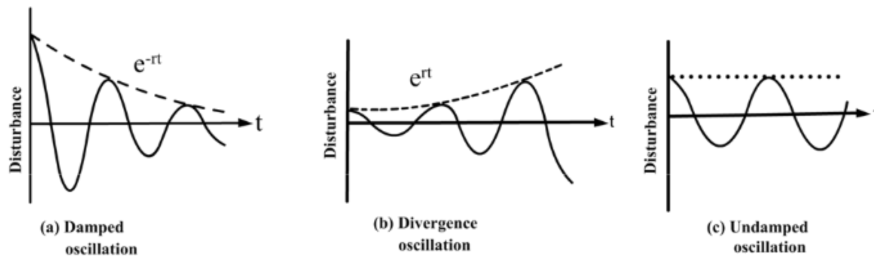


Figure 2: These give examples of dynamic stability, assuming the horizontal axis is time and the vertical axes are displacement or angle from equilibrium. The oscillation with the e^{-x} damping exhibits a damping ratio large enough to drive the system towards equilibrium, indicating positive dynamic stability. As can be inferred for similar reasons, the middle image depicts negative dynamic stability, and the last depicts neutral dynamic stability [4]. (Ogata, K., 2010, *Modern Control Engineering,* 5th ed., Prentice-Hall, India.)

A Phugoid is the summation of the above; it graphically displays the oscillation of a craft in an axis over time. Analysis relating to frequency, damping ratio and amplitude are common, described further in Data Representation (section 1.6).

1.4 Assumptions and Applications

A full analysis of static and dynamic stability for a craft does not only take into consideration the effects if briefly disturbed in straight and level flight; there are many flight conditions a craft's stability is tested in. Even this case has restrictions and limitations, for example if you hit any plane hard enough it will fall from the sky. A full analysis can be used to size control surfaces to correct itself in common precarious situations where the plane could fall from the sky. Tail spin, dutch roll, vibration, and takeoff with a crosswind are common scenarios a craft is designed around [5].

Static stability requirements create a few guidelines for the dimensional design of a plane. Mentioned in the Derivation section (Chapter 2), these can be used for initial dimensions, while the full stability analysis tells how a craft's configuration performs and indicates alterations that can improve stability. A significant assumption going in to the dynamic analysis is that the craft in question can leave the ground successfully, or that it has enough desirable dynamics to be statically stable in air in the first place.

Stability is quantified using a few different methods, and it is not always beneficial to have very high stability. Chapter 4 explores the cases against having a stable plane.

Another large limitation of the analysis to follow is the craft being analysed. The examples provided by the Union College SAE Aero team are all rectangular, top-winged, conventional tail crafts, an example of which seen in figure 3. Its wing and horizontal stabilizer are rectangular, and the taper of its vertical stabilizer is not evaluated in this calculation. Other plane configurations will alter stability differently, changing the magnitude or location of a force or moment about the center of gravity. Therefore even if this process applies to any craft, this analysis and code (presented in section 2.4), if unaltered, applies only to the craft configuration in figure 3. Additionally, an underlying assumption is that the craft stays within a margin of 10 degrees above or below its equilibrium, as the farther the craft diverges from an angle of zero, the less accurate the approximation is. At larger angles, forces caused by the fuselage start to become more significant, and data taken only from static wing analysis start to omit more important factors.

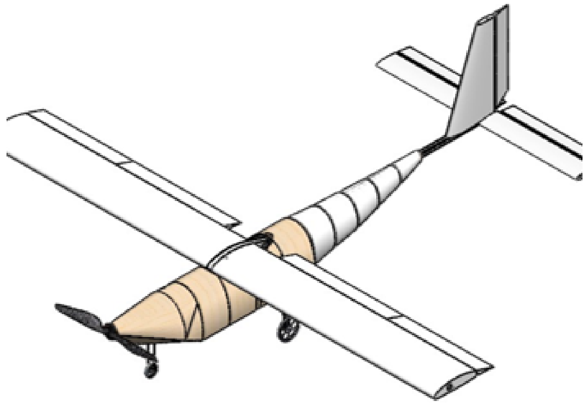


Figure 3: Union College's SAE Aero plane design from the 2020 Advanced event, used for static analysis in the Derivation section (Chapter 2) [6]. (Bruno, A. and Caro, K. and Khan, M. and Landreth, R. and Santos, Joana, A., 2020, "Union College Flying Dutchmen Advances Design Report," Union College, Schenectady NY, p. 27.)

1.5 Methods of Quantifying Stability

There are a few major methods of quantifying the stability of an aircraft. Static analysis often includes control derivatives which monitor the change in moment about the center of gravity with respect to angle of attack. The conditions for static stability are that the moment is zero about the center of gravity, and the derivative of this moment with respect to the angle of attack being negative. This means as the plane tips up from equilibrium, the plane induces a negative moment and corrects the plane back towards equilibrium. If it tips down, a positive moment corrects the plane. The notation for this is shown in equation 1 [7], following the List of Acronyms on page vi.

$$C_m = 0 \quad \text{and} \quad \frac{\delta C_M}{\delta \alpha} < 0 \tag{1}$$

Dynamic stability is often measured, and this is useful in presenting how one plane configuration will function. If instead a variable's effect on the stability can be monitored (all else unchanged), its designed value can be altered with the knowledge of what it will do to the craft. An example of this type of analysis includes the Roots Locus method, which will require some knowledge of Laplace transforms and control systems practices provided later on. It basically derives a characteristic equation governing the craft's dynamics, and for a craft it looks at the roots of this equation for all values of a certain variable [4].

Additionally, the speed at which the craft can turn about its axis is often calculated, to describe how easily the pilot can adjust the plane. This aids in dimensioning ailerons and elevators, and is also applied to rudders to keep the craft in the right orientation in the event of a tailspin or high crosswinds in takeoff.

1.6 Data Representation

Each methods has different ways to represent this data graphically. Static analysis often involves simple plots depicting the moment about the center of gravity vs the craft's angle of attack such as figure 4, or moment diagrams used for balancing moments. For the design of an aircraft, horizontal moment diagrams are very useful in placing the center of gravity. In static analysis, these can be better used to visualize the components of force that the wing, tail, and fuselage contribute while at an angle.

Dynamic stability has many ways of being depicted, and this analysis often informs the designer how much to alter dimensions for stability, or how stable the craft is. Because of this, methods like the Root Locus analysis (an example of which depicted in figure 5.) are often viewed with respect to all possible values of one variable [4]. It shows how much a

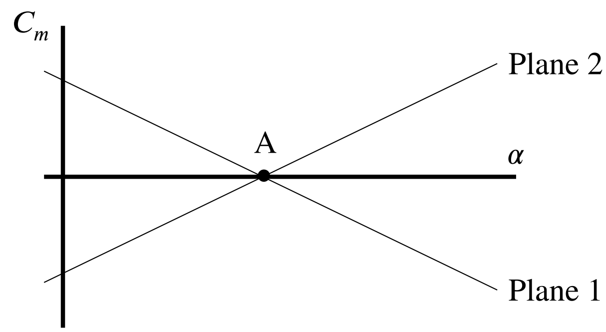
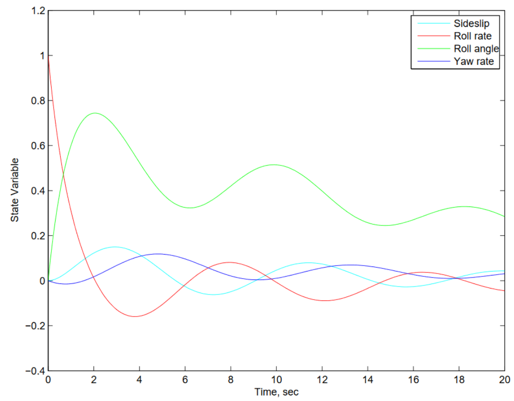


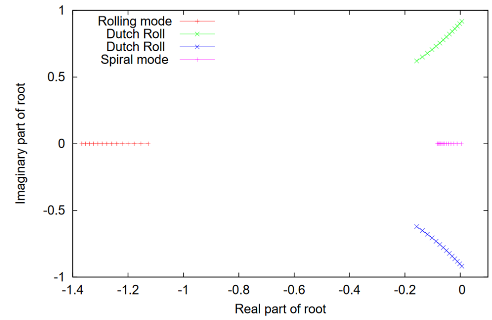
Figure 4: Example Moment Plot for Static Stability presenting Moment (C_m) vs Angle of Attack (α). Point A indicates equilibrium, Plane 1 shows positive static stability (angles above equilibrium express a negative moment pushing it back down to A), and plane 2 shows negative static stability.

given variable affects the craft's stability based on where it is on the plot, and can indicate what circumstances make the plane unstable [8].

Phugoids show the oscillation of the craft about an axis with respect to time, as mentioned earlier, and graphically shows other important data. The information this offers relates to how fast the oscillations are, how quickly they'll die away, and their relative magnitude [9]. For example, a plane oscillating too fast disrupts the boundary layer of airflow over the wings and subtracts from lift produced [1]. The value of the damping ratio for a given system (ζ) is a good way to remember the difference between positive, neutral, and negative dynamic stability, as those are relative magnitudes of the damping ratio for those scenarios [4]. The damping ratio is a dimensionless number indicating how fast the motion in the system decays towards equilibrium for a second order differential equation. An example of this is presented in figure 5, showing a phugoid under a few different conditions for a Boeing 747.



(a) Boeing phugoid example, after disturbances showing positive dynamic stability.



(b) Roots locus for Boeing 747, depicting what values cause negative, positive, and complex roots in the system response, indicating stability.

Figure 5: Dynamic Stability Visualization Examples for Boeing 747 [7]. (Caughey, D, 2011, Introduction to Aircraft Stability and Control, Cornell University, p 94 from https://courses.cit.cornell.edu/mae5070/Caughey_2011_04.pdf.)

Chapter 2

Deriving Longitudinal Static Stability

2.1 Static Stability: What Goes In and What Comes Out

A static stability calculation starts with a moment balance. As mentioned in the last section, equation 1 provides the requirements for static stability; the moment about the center of gravity must be zero, and the derivative of the moment with respect to the angle of attack must be negative. Moment diagrams for the center of gravity at zero angle of attack, with a positive angle of attack, and with a negative angle of attack can be evaluated to roughly determine the moment and its derivative about its equilibrium point.

To do this, a diagram is an easy way to pair the major geometry of a plane with the various forces and moments. The plane will conform to the assumptions given in section 1.4. Figure 6 shows a plane at a zero angle of attack for initial analysis, with subsequent figures pertaining to the craft at different angles. This is the Aero 2020 advanced craft design, known to fly with reasonable stability but not thoroughly tested enough to optimize dimensions [6]. Static analysis will confirm flight capability and inform possible decisions of dimensions that can be changed to adjust static stability.

2.2 Deriving a Model

2.2.1 Longitudinal Static Stability Moment Summation

As with any static analysis, the place to get started is with a force body diagram. specifically, summing the forces and moments around the center of gravity. This creates a general model that evaluates the dimensions of a real plane in subsequent sections. The center of gravity being the focal point means that its weight does not produce a moment, simplifying the summation. The forces of lift are placed at the **aerodynamic center**(AC) of the wing and tail airfoils, being a point where one force can be added to compensate for the full pressure distribution over the airfoil of the wing/tail, and where the pitching moment coefficient does not vary with the angle of attack. That location is typically approximated at $1/4$ of the way down the chord [2]. The pitching moment coefficient acts in a similar way to the lift and drag coefficients, scaling the moment which is also proportional to airspeed, density, and wing surface area. A symmetrical airfoil has zero moment about the aerodynamic center [8]. The aerodynamic center can also be found for the entire plane as well, but this example will use both individually.

In figure 6, components of lift are placed at the solid dots marking the aerodynamic centers of each individual airfoil. The center of gravity is typically marked by the black and white circle seen in the figure, and measured by summing the moments caused by only the masses of each item on the craft. Equations 3 to 7 are a fairly verbose version of the analysis, expanded to show separate moments acting on the craft, then breaking each moment into its components.

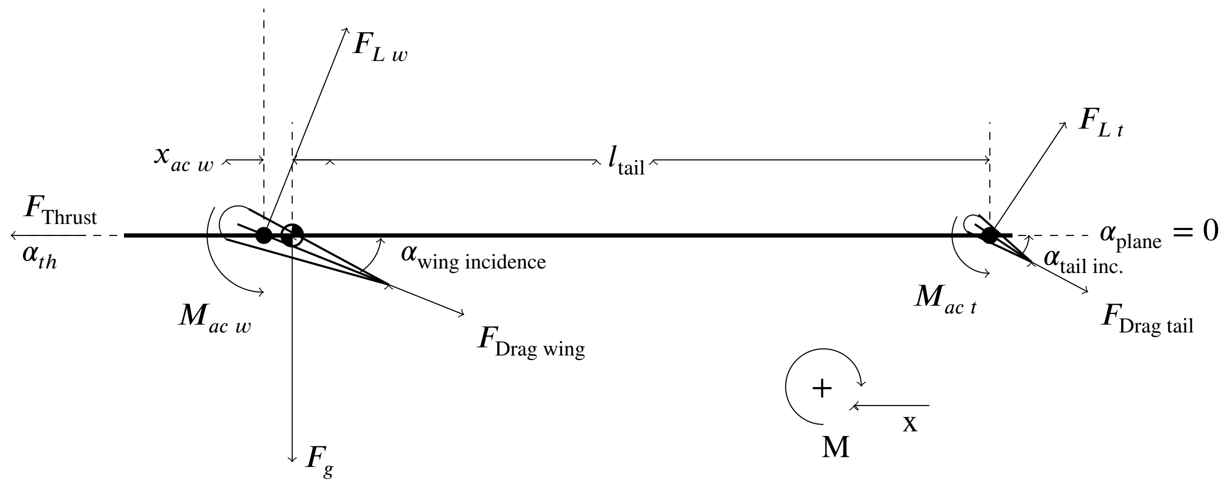


Figure 6: Moment Plot for Static Stability presenting at $\alpha = 0$, and including the possibility for the wing and tail to have an angle of attack relative to the craft, an angle of incidence. The direction of positive x and positive moment are indicated in the lower right, and variables are explained in the List of Acronyms.

A very important thing to note is that the vector of thrust can add a moment if it is put at an angle. This can angle the plane and add an angle of attack to the wing and tail, changing performance drastically if it is not anticipated. For this example, it is assumed to be directly in line with the center of gravity. In addition, the wing and tail are pictured at an angle in figure 6. This possible angle of attack with respect to the aircraft's centerline is called an **angle of incidence**. This is often done to cause the craft to have an altered angle of attack while in air, changing its lift coefficient and potentially adding lift. In reality, a very important context to think about an angle of incidence in is the takeoff conditions of the craft, where any craft wheels tilt the craft in addition to the angle of incidence, changing its lift coefficient and lift produced. If the craft is thrown for takeoff, this is not as significant an issue. **Assumed below** in equations $3-7$ is that the wings and tail are at a **zero angle of incidence**, so α_{wing} and $\alpha_{\text{tail}} = 0$, with the direction of positive moment noted as clockwise in this orientation, or counter clockwise if looking at the plane facing right. Because tail airfoils are usually symmetrical, the moment about the aerodynamic center of the tail is zero.

$$C_{m\ cg}(0°) = M_{\text{wing}} - M_{\text{tail}} \ = \left(F_{L\ \text{wing}}\ x_{ac\ w}\right) - M_{ac\ w} - \left(F_{L\ \text{tail}}\ l_{\text{tail}}\right) \tag{3}$$

$$C_{m\ cg}(0°) = \left(C_{L\ \text{wing}}\rho\frac{v^2}{2}A_{\text{t wing}}\ xac_w\right) - M_{ac\ w} + \left(-C_{L\ \text{tail}}\rho\frac{v^2}{2}A_{\text{t tail}}\ l_{\text{tail}}\right) \tag{4}$$

$$M_{ac\ w} = C_{Mc\ ac}\frac{v^2}{2}\rho b_w c_w^2 \tag{5}$$

$$C_{L\ \text{wing}} = C_{L\ \text{wing}}(\alpha_{\text{wing}}) \qquad C_{L\ \text{tail}} = C_{L\ \text{tail}}(\alpha_{\text{tail}});$$

$$\alpha_{\text{wing}} = \alpha_{\text{plane}} + \alpha_{\text{wing incidence}} = 0 + 0 \qquad \alpha_{\text{tail}} = \alpha_{\text{plane}} + \alpha_{\text{tail incidence}} = 0 + 0$$

(Noting that coefficients are functions of α)

Equation 6 factors out the airspeed and the air density, and separates the wing and tail areas into the product of their chord (c) and their span (b), usable for a rectangular wing and horizontal stabilizer as per the assumptions listed in section 1.4. Also, it incorporates equation 5 to simplify it.

$$C_{m\ cg}(0°) = \rho\frac{v^2}{2}\left[\left(C_{L\ \text{wing}}(0)c_w b_w x_{ac\ w}\right) - \left(C_{L\ \text{tail}}(0)c_t b_t l_{\text{tail}}\right) - C_{m\ ac\ w} b_w c_w^2\right] \quad (6)$$

A full equation without the assumptions of the craft angle of attack, angle of thrust, and angle of incidence is presented in equation 7. It follows the same procedure as the above, with a few more moments and angled components :

$$C_{m\ cg}(\alpha) = \rho\frac{v^2}{2}\Bigg[x_{ac\ w} b_w\left(c_w C_{L\ w} Cos(\alpha_{I\ w} + \alpha_{\text{craft}})\ -\ t_w C_{D\ w} Sin(\alpha_{I\ w} + \alpha_{\text{craft}})\right)$$
$$-l_t b_t\left(c_t C_{L\ t} Cos(\alpha_{I\ w} - \alpha_{\text{craft}}) - t_t C_{D\ t} Sin(\alpha_{I\ t} + \alpha_{\text{craft}})\right) - C_{m\ ac\ w} b_w c_w^2\Bigg] + F_{Th} x_{\text{motor}} Sin(\alpha_{I\ \text{Motor}}) \quad (7)$$

This model can be improved by making the thrust force a function of airspeed, adding a term for the moment about the AC of the tail in case the tail airfoil is not symmetrical, and by adding other components of lift and drag related to skin friction and the body of the plane. This approximation breaks down at larger angles in part because the fuselage contributes more and more to forces of lift and drag as more of it is exposed. A term for the fuselage can be added that approximates several flat plates, relating angle of the surfaces to lift and drag components, and a few other variables correlated in textbook fluids tests [1]. Additional improvements include that lift and drag forces are not constant over the wing span and can be approximated more accurately [10], and several other components of drag caused by skin friction and wing tip vorticies can be added to increase accuracy. Even manufacturing changes the amount of surface drag due to friction and the accuracy of copying the exact shape of the airfoil changes its effect. This clearly leaves room for improvement, but equation 7 provides a good idea how the plane will react under cruise conditions.

2.2.2 Longitudinal Static Stability Moment Derivative

Next, the same system will be analysed at a positive angle of attack, $+d\alpha$, in figure 7 described by equation 8. Although the equation does not change significantly, moment arm distances and force vectors are changing with respect to the angle. This will, for example, shorten the major moment arm between the tail and the CG, but will also decrease the component of force directly upwards produced by the wing. The surfaces of the fuselage will start to add to the lift and drag more significantly at an angle, though this will impact lift sooner and drag less if the nose is conical.

$$C_{m\,cg}(d\alpha) = \rho\frac{v^2}{2}\left[x_{ac\,w}b_w\left(c_wC_{L\,w}Cos(d\alpha)\ +\ t_wC_{D\,w}Sin(d\alpha)\right)\right.$$
$$\left. +\, l_tb_t\left(c_tC_{L\,t}Cos(d\alpha)\ +\ t_tC_{D\,t}Sin(d\alpha)\right)\right] + M_{ac\,w} \quad (8)$$

This is similar to equation 7 but replaces angles with the increment $d\alpha$, does not incorporate thrust (which is still assumed to be in line with the CG), and includes the wing moment separately for visibility. Lastly, the same system will be analysed with the whole craft at a negative angle of attack, at $-d\alpha$ in equation 9.

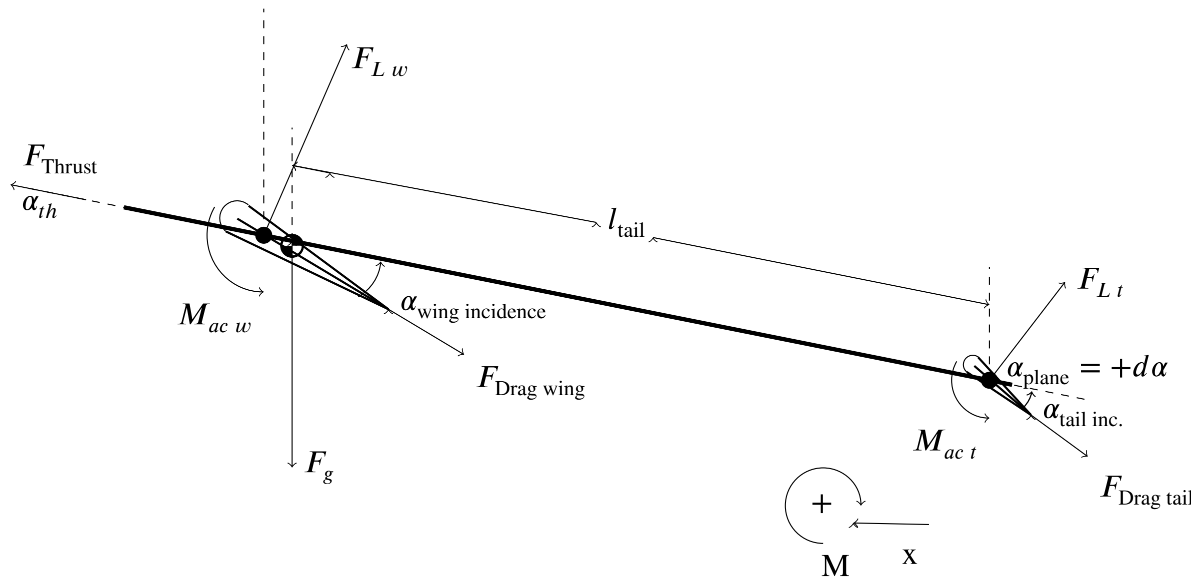


Figure 7: Moment plot for static stability presented at $\alpha = d\alpha$ with all other dimensions the same as figure 6.

$$C_{m\ cg}(-d\alpha) = \rho\frac{v^2}{2}\Bigg[x_{ac\ w}b_w\Big(c_w C_{L\ w}Cos(-d\alpha)\ +\ t_w C_{D\ w}Sin(-d\alpha)\Big) + l_t b_t\Big(c_t C_{L\ t}Cos(-d\alpha)\ +\ t_t C_{D\ t}Sin(-d\alpha)\Big)\Bigg] + M_{ac\ w} \quad (9)$$

With these three points, a simple plot will show the relationship between the plane's angle of attack and moment so that the second requirement, the derivative of the moment about the CG can be evaluated. Equation 10 may be visually difficult to evaluate as positive or negative. To aid in this the dimensions and airfoil relations of the 2020 Union College Aero Advanced plane can supply more meaningful information, given in table 1.

$$\frac{\delta C_M}{\delta\alpha} = \frac{\frac{C_m(0)-C_m(-d\alpha)}{d\alpha} + \frac{C_m(d\alpha)-C_m(0)}{d\alpha}}{2} = \frac{C_m(d\alpha) - C_m(-d\alpha)}{2d\alpha} \quad (10)$$

This is an approximation for the pitch control derivative. There are more complicated ways to break down geometry and find this derivative including smaller $d\alpha$ or control derivatives, but this is a reliable stick-fixed method and can be done after a common statics or dynamics class.

2.3 Applying the Model

These equations are slightly abstracted away from a specific physical craft, so for this physical analysis these abstractions are given sample values relating to the 2020 Union College Aero Advanced plane. Concerning a positive and negative angle of attack, $\pm d\alpha$, the deviation from a zero angle of attack will be set not at enough for the wings to lose lift ('stall,' usually in the order of $\pm 10°$), but close enough to get a wide range of values: $\pm 8°$. Testing a range of values for this interval reveals how linear the relationship is as well.

Major Dimension	Value
Wing Span	123.25"
Wing Chord	16.5"
Wing ac to Tail ac	70.1"
Tail Span	55.0"
Tail Chord	9.22"
Wing Incidence Angle	3°
Tail Incidence Angle	0°
Thrust Incidence Angle	0°
CG Location	5.5" from Wing front

Table 1: 2020 Aero Advanced plane major dimensions for use in static stability model [6].

In addition to this, the moments generated at each airfoils are needed, as is the maximum thickness of the airfoil, because that cross sectional area is what the drag force scales with. The lift and drag coefficients with respect to the angle of attack are also needed, provided in figure 8. Not shown are those for the tail, though for this plane the tail uses an e168-il

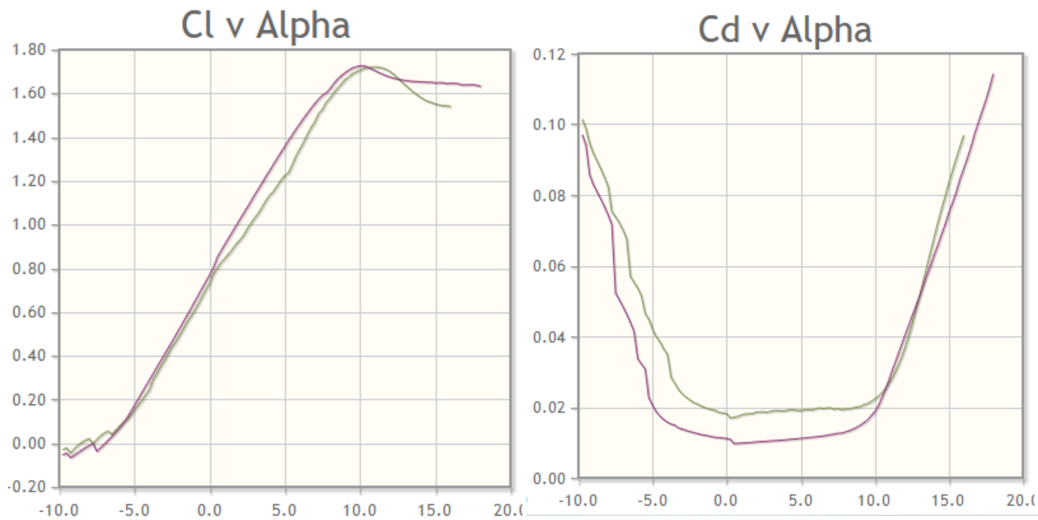


Figure 8: Lift and drag coefficients vs angle of attack for la-203a airfoil. The vertical axis is the coefficient value and the horizontal axis is the angle value. The green and purple lines indicate different Reynolds numbers, of 200,000 and 500,000 respectively, relating to the turbulence of air flowing over the airfoil. For a plane with 16.5 inch airfoils moving at 30 miles an hour in air, the Reynolds number will be closer to 350,000 [11, 12]. ("Douglas LA-203a Airfoil," http://airfoiltools.com/airfoil/details?airfoil=la203a-il, and "E168 (12.45%) (e168-il)," http://airfoiltools.com/airfoil/details?airfoil=la203a-il.)

symmetrical airfoil with data well known and in many databases, a very easy one being airfoiltools.com [11, 12].

Inserting the measurements from table 1 into the model generated in equation 7 produce the following values for the moments and derivative. The plots in figure 8 produce values of lift, drag, and moment coefficients for each of the angles usable for the model.

$$C_{M\ cg}(0°) = 37201\ lbf * in$$

$$C_{M\ cg}(2°) = -31102\ lbf * in$$

$$C_{M\ cg}(0°) = 102650\ lbf * in$$

$$\frac{C_{dM\ cg}}{d\alpha} = -13376\ lbf * in \text{ per degree}$$

Using the derivative and the moment about the center of gravity at a zero degree angle of attack, the equilibrium angle at which this plane remains stable is 2.7°. The script in the following section is a fast way to get the same results quickly and with out common errors, and can be added onto for a more accurate model if so desired.

2.4 Script

Appendix A shows a MatLab script that takes the dimensions of a plane and does the same analysis as above in section 2.2. This section will go over the inputs and outputs of this script, how to interpret them, and how they are manipulated in conformance to the characteristics of a stable craft.

The top segment shows a plethora of variables with the values of this plane entered in. The output was mentioned in section 2.3, but in essence the script accomplishes exactly what equation 7 predicts when the plane lies flat, and $d\alpha$ degrees on either side of it. The functions before the main calculating segment are used to approximate the coefficients of lift and drag. These are not continuous functions, just peace-wise approximations of the behavior of these coefficients as given by the database airfoiltools, where they were rigorously tested alongside over a thousand other differently shaped airfoils [11, 12]. For the Douglas la-203a airfoil, the coefficient of lift is approximated using equation 11, a rough approximation comparable to the C_d vs α plot in figure 8.

$$
C_d(\alpha(\text{in degrees})) = \begin{cases} 0.000140654 - 0.0103897\alpha & \text{if } \alpha < -2 \\ 0.0193 & \text{if } -2 < \alpha < 10 \\ 0.01282\alpha - 0.1098 & \text{if } \alpha > 10 \end{cases} \tag{11}
$$

To use this script for a different plane, **several things will need to be changed.** The functions concerning the lift, drag, and moment coefficients in the five functions before the "Longitudinal_Stability" function are unique to this wing/tail airfoil combination. These will have to be changed to make them resemble the wing airfoil in use, though other symmetric tail airfoils have fairly similar behavior to this. Additionally, the input dimensions in the first block of text must reflect the craft in question. **The outputs in order** are the moment about the center of gravity at a zero angle of attack, and angle of attack $d\alpha$ (default 2°) above and below in units of pound force inches. The fourth output is the moment derivative, and the fifth is the angle at which static stability is achieved based off the moment at zero angle of attack and the derivative. Lastly, a string evaluating the results is printed. There are a few default values in the equation that don't significantly affect the outcome but should be known. The default plane velocity is 25 miles per hour, the air density equivalent to $1.225 kg/m^3$, and the change in angle is 2°.

A very useful tool while designing a plane is a general idea of what altering one dimension will do for the craft. Everything has drawbacks, and looking at the model above (and represented using the code in appendix A) the effect of changing a dimension can be related back to how it affects the craft. Again, changing dimensions will always have both benefits and drawbacks, for example increasing tail length necessarily means increasing support to avoid failure, etc., so this section only explores a change's effect on stability performance.

The largest effect on the angle of equilibrium and the restoring moment is not the length of the tail's moment arm or the size of the wing, but the placement of the center of gravity. Table 2 shows a sensitivity analysis for this model as it pertains to the Aero 2020 plane. The partial derivatives of the output values with respect to dimensional input variables clearly show the largest change is due to changing the angle of incidence for the tail. Interestingly enough, this does not change the restoring moment derivative, but does change the equilibrium angle. This means the plane will self-correct very similarly, but the equilibrium angle the plane has static stability around shifts. That makes sense when considering the job of the elevator is basically changing the angle of the horizontal wing piece to angle the plane up and down. The largest change in the moment derivative is from increasing the chord of the tail, which lowers the moment derivative and causes increased pitch correcting moment. This is a good method of analysing an aircraft to alter the existing dimensions and receive instant feedback on what it will do to flight.

δ Output	Wing Chord	Tail Chord	AC_{wing} to CG	AC_{wing} to AC_{tail}	$\alpha_{I\ w}$	$\alpha_{I\ t}$
δ Moment at 0°	3880	0	8662	0	1262	-14879
δ Moment at $+d\alpha$°	3904	-12784	14202	-1681	-1065	-14348
δ Moment at $-d\alpha$°	2510	12790	1650	1680	2420	-14480
δ $\frac{dMoment}{d\alpha}$ 0°	87	-1598	785	-210	-218	8
δ Equilib. Angle	$-0.305°$	$+0.293°$	$-0.854°$	$+0.042°$	$-0.048°$	$+1.104°$

Table 2: A sensitivity analysis displaying how significantly and how a dimensional change affects the moments of the craft in flight. Each parameter was increased by 1 inch or 1 degree, and the difference in values were recorded before being reset. The moment units are lbf*in, and the angle units are degrees.

The numerical outputs are displayed in the console, but the program also immediately graphs the moment over a range of -10 to 10 degrees for the parameter values entered. This is a very fast way to visually check for positive, negative, or neutral static stability. Figure 9 matches plane one from figure 4 in the two conditions for static stability. The fact that a point exists in a reasonable angle range with zero moment and a negative moment derivative confirms that this plane is statically stable.

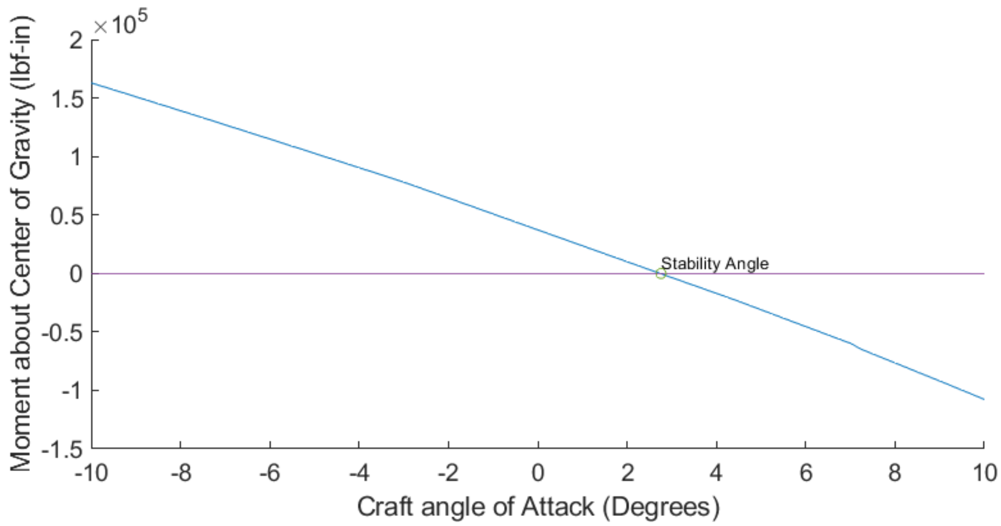


Figure 9: Union College 2020 Aero Advanced plane showing at its equilibrium angle (2.7°) it has a negative moment derivative with respect to angle, meeting the conditions of equation 1.

2.5 Roll/Yaw Stability and Common Flight Instabilities

The derivation above goes through the stability equations concerning pitch, which does leave out two axes of movement. This is because longitudinal stability is, arguably, the hardest to correctly evaluate and the most often done incorrectly. This also means that there are characteristic mistakes associated with pitch instability that can be diagnosed after seen in flight. Low longitudinal stability can just mean that a small input greatly alters the trajectory of the plane, so much as to lose control all together. However, common mistakes with longitudinal static stability can often cause little time to evaluate further flight. If slightly off the plane could tend to pitch down or up with no input, but with a more significant problem, upon takeoff (if able to take off), the craft will either arc directly up and crash, or directly down and crash.

Yaw and roll static stability are more intuitive. If the craft is designed symmetrical with internal and external components mirrored, then roll stability exists in some capacity. This can be increased by lowering the center of gravity vertical on the craft. This includes putting the wings higher on the fuselage, or even angling the wings up from the fuselage to add dihedral effect. Giving the wings a dihedral angle causes the plane to center itself in flight and provides some resistance to rolling, tilting the plane causing one wing to tilt farther up on the outside of the turn, with the opposite effect pairing to cause the outside wing to produce less lift than the wing inside the turn radius. Yaw static stability provides resistance from the plane spinning like a top. This is rarely seen, as a symmetrical plane with the vertical stabilizer in line with the center of the craft is enough to keep the plane flying straight, as long as the vertical stabilizer is large enough and the rudder is straight. There is little downside to over sizing the vertical stabilizer, besides adding weight to the tail and moving the center of gravity back.

Characteristics of a plane with low yaw and roll stability include erratic turning and rolling, very easily thrown off course. A dutch roll is where the tail of the plane in flight moves in a circular motion, and this often indicates either adverse conditions or insufficient tail horizontal and vertical surface areas. Other difficult situations like tailspins, crosswinds in takeoff causing issue, and wing fluttering are due to lack of control, benefited by expanding the related control sure size. A takeoff in a crosswind requires a large enough rudder to keep it pointing in one direction, and a common way to size the rudder is to simulate the control needed to break out of a tailspin. Wing flutter is caused either by the wings not be affixed solidly enough, or more often by slop in control surfaces. Slop refers to the unwanted movement of control surfaces if they aren't attached firm enough, and the flutter can happen in multiple axes, greatly reducing stability and control authority.

Chapter 3

Dynamic Stability: What Goes In and What Comes Out

There are, as mentioned earlier, many methods possible to characterize the dynamics of a plane in motion, and many scenarios a plane experiences that it must be able to recover from. The roots locus method is very useful in the design process, and for determining how viable a design is. Unfortunately, it takes an understanding of control systems usually found in a Dynamics of Physical systems class (for example, Union College's MER-322) to derive from hand. A brief description of its derivation and uses is presented in section 3.3.

Alternatively, the phugoid analysis as presented in section 1 can classify aircraft stability in terms that can be directly compared with other crafts, including crafts outside its scale of size. This analysis stems much from information taken from a dynamics class, and although the geometry of the craft can lead to some complex relationships, if followed diligently the reference [9] can be very useful in leading through the relations and important equations in sections 1, 2 and 3. The use of computational methods leads to dynamics equations in a slightly different format, relying much on matrices and integration techniques to approximate the craft behavior through time.

The most intuitive seeming dynamic stability 'number' is the amount of time it takes to turn the plane through a certain angle on one axis, for example timing 3 seconds to turn 60° in yaw, roll, or pitch. That statistic is why many regulations are put in this format, with higher time indicating a more stable craft and lower times indicating a more maneuverable craft.

3.1 Dimension for Turn Timing

After a plane has some measure of static stability and can get into air, the maneuverability becomes important to the pilot and its overall stability. Dynamic stability analysis methods often include control flap analysis (ailerons, rudders and elevators) [5], but a way to dimension control surface sizes around a certain stability is to calculate how much of a moment it must produce to turn the plane through a given angle in a given time.

Giving a time that the plane must tilt, say 45° in the roll, is an intuitive standard, and can serve as a guideline for dimensions after the non-moving parts of the plane are set. Much of the analysis for such a technique is based off, again, summing the moments including those made by the control surfaces deflecting. This time the analysis uses equation 12, the angular relative of $F = ma$. The x axis denotes the roll axis, used for designing ailerons. Assuming zero initial angular velocity:

$$\sum M = I_{xx}\ddot{\theta} \;\; \Rightarrow \;\; \frac{\sum M}{I_{xx}} = \ddot{\theta} \;\; \Rightarrow \;\; \frac{\sum M}{I_{xx}} = \frac{2\Delta\theta}{\Delta t^2} \tag{12}$$

This must incorporate both the start and stop distances of the control surface, the angle they deflect, and the fraction the total airfoil chord they take up. The theory involved isn't overly complicated, but the system is fairly complex and is fairly involved to incorporate everything. Thankfully, a script was also written for the purpose of sizing ailerons during the development of the 2020 Aero advanced plane [6]. For sizing the elevator and aileron, there are example problems in Sadraey's text as well. If these are used, as a precaution always make the total angle the control surface can swing through (throw) larger than necessary for a first flight, up to 45° each way. Much more of a throw and they lose effectiveness, but having more throw than necessary in an untested craft is a good precaution in case unexpected behavior requires fast action. Recommended for elevator and ailerons is deflection up around 25°, and down around 20° from center [5].

3.2 Phugoid Analysis

The method primarily taught in aerodynamics classes for dynamic stability involves a matrix equation of inputs and control derivatives used to calculate the output plane response, often in a phugoid format. The equation structure is also textbook control systems or dynamics of physical systems [4], though the difficult part is not the structure, but how to calculate the control derivatives for each control surface on the plane. In section two, the sensitivity analysis included changing the tail angle by one degree and noting its effect on the moments, which is *an* approximation of the control derivative for the elevator, but only if the angle the elevator moves can be equated to an effective angle change of the whole horizontal tail.

Classes from MIT [2] and Cornell [7] both describe similar methods of phugoid generation, though as such the courses build on themselves and it takes an amount of back tracking to understand and find notations and calculations. Phugoids are created with dynamics equations and an initial condition; usually this initial condition is the perturbation that creates the correcting oscillations. Sample initial conditions involve a craft total angular displacement from equilibrium of one degree, movement of the elevator/aileron/rudder by 1 degree, or some amount of initial angular velocity/acceleration. The angles are small so that it stays within the bounds of the model.

The basics of the matrix equation system used is taking a series of differential equations and inserting them into matrices, so that computer algorithms can solve them very neatly and integrate over a time step, producing many points that would approximate the real behavior. The static analysis done earlier produced an equation for the Aero 2020 plane's pitching moment at a given plane angle. Using $M(\theta) = I\ddot{\theta}$, this can be made into a rough approximation of the moment with respect to time. Using the Runge-Kutta 4 integration algorithm in a self made code, the phugoid approximation produced is displayed in figure 10.

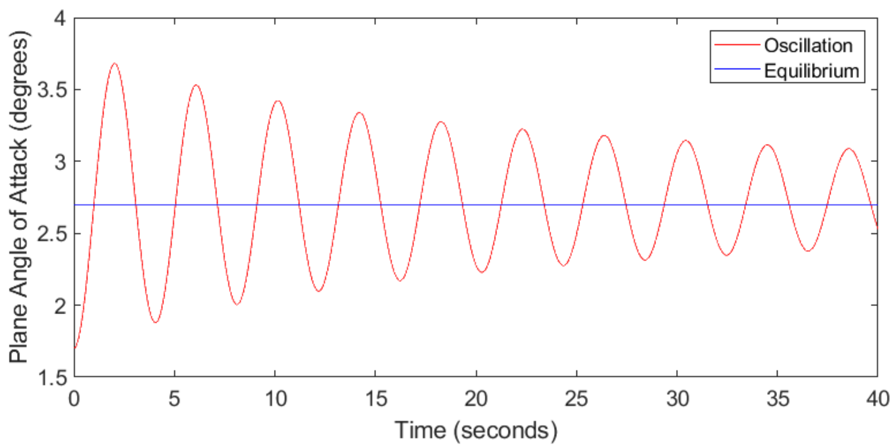


Figure 10: This displays a rough phugoid approximation of the Aero 2020 advanced plane displaced one degree from its equilibrium point found earlier, 2.7°, without inputting this information. This confirms the equilibrium angle, and when the tail area is increased the oscillation decays faster, correlating to a higher damping ratio and a more stable craft.

This is also a stick-fixed analysis, done without moving the control surfaces and only modeling the attenuation caused by the drag of moving the wing and tail. Many phugoids model the use of elevator to dampen the oscillation much faster, so this approximation shows the oscillation propagate for a relatively significant amount of time in flight.

3.3 Roots Locus Method

Roots locus method is very helpful in educating dimensional design, though requires knowledge of the Laplace transform and transfer functions to produce this from scratch. Basically, a system with a known set of forces and relations can be described by a differential equation, in this case as a function of angular displacement, velocity, and acceleration. This function could then be put into the complex frequency domain using pre-calculated methods, like the Laplace Transform [4]. The transfer function then is the Laplace transformation of the output as a ratio of the Laplace transform of the input, assumed for zero initial conditions and only for linear systems. A transfer function characterizes a system, so most any reasonable input can be directly converted to the system output. This analysis essentially makes a transfer function for the system, and inputs one dimension as a variable instead of the value on the craft. At this point it can be expressed as a ratio of polynomial equations. Then, it evaluates the roots of the transfer function, the x intercepts of the denominator. This gets fairly complicated to do by hand, so is often done computationally. In fact, it gets more and more difficult the more terms there are in the denominator.

Another quicker and similar method that does not make a root locus plot uses Routh's Stability Criterion [4]. This calculates the characteristic equation of a system, but does not find the roots of it. This makes it much simpler for more complicated systems, and that much faster as well. Instead, this method keeps the characteristic equation as a ratio of two polynomials, and evaluates the constants to predict whether the roots could contain complex terms or real terms. It is desirable for the characteristic equation denominator's roots (poles) to be negative, because that would indicate the system is bounded properly to produce reasonable values, and will have reasonable behavior [4].

Chapter 4

Alternative Methods and When Stability is Bad

There are other methods of achieving the preceding results. This includes software, testing, and simplified dimensional relations ('rules of thumb'). Each of these, including the previously explored analytical models, have their pros and cons in the design process. The main downsides of an analytical model include the informational barrier that prevents its use. It is also difficult to initially design a craft around dynamic stability, though static stability can inform a dimensional designing. These models can be used reliably to both determine how air worthy a craft is, and adjustments that manipulate this stability.

4.1 Stability Confirming Software

The subject of useful software is difficult, as they are constantly changing and improving. All in all, unless it is industry grade, a sense of doubt is healthy. One such military program often used is United States Air Force Stability and Control Digital (**DATCOM**). Many academic papers rely on DATCOM [9] for static and dynamic analysis. It is public domain, and where they still incorporate unknown assumptions, some research could make this a useful tool. There can be software specifically made with the assumptions pertaining to one craft that will work for that craft. However, unless complete knowledge of those assumptions and the software is known, significant mistakes are common. For example, coded examples in textbooks are right next to the theory and are explained well, being fairly reliable even if personal code is based made from the same source material. Finding good software on the internet is difficult without doing thorough research. The most important thing to consider is that many software like SolidWorks simplifies conditions, and is limited in use for fluids applications.

In the struggle to provide evidence that a design functions properly, often students and those with access try using CFD software. A senior project by Kevin Caro ('20) more closely explores the application of CFD software in the design of an aircraft [10]. His major conclusions comprise both that to effectively use CDF and software, knowledge of fluid dynamics, computational methods, finite difference algorithms, mesh generation and analysis, boundary conditions, system specifies and more, and that CFD software should not be used during the design process [10].

4.2 Physical Testing

Physical testing, if possible, is the most accurate way to determine the stability of a plane. If the resources exist to have a plane ready to fly, using the flight to acquire as much data as possible is very important. Additionally, flying one time means fairly little, as even if the flight is perfect, different flight conditions and wear can be the difference between a repeatable result and a pile of wreckage.

The repeatablilty of a result is fairly important in testing. In any environment the conditions are constantly changing, and no takeoff is the same. It also leaves little time to watch any problems arise, or parts to wear out and not in their optimal conditions, being more realistic. In this way a failure is a more useful flight if that problem is known. There have been many examples of stability problems that arose quickly because they are drastically wrong, and smaller problems that have persisted because they are not observed until it is too late. These are fairly obvious to the eye and can be avoided, even if they halt progress if a manufactured craft fails in these ways. A good flight practice is to repeat a test several times and be able to duplicate the results without problems before being satisfied with a design.

Stability derivatives can easily be measured with an on-board 3-axis accelerometer. The recorded date can show how fast the craft's inertia turns and catch oscillations that can be evaluated much better than watching the craft during flight. Moving the control surfaces then can produce a measurable response, and the physical dimensions and materials of the craft produce the mass moment of inertia that an input caused to produce the recorded response. Not only can the static stability be analysed, but recording the system response to stimuli over time does the added benefit of recording over time the system response to stimuli; this is a very easy way to test the plane in specific conditions for its dynamic stability. This means data analysis may be tedious or time intensive. Physically testing a design is inherently one of the most accurate ways to collect data on how the craft responds, though only if the resources exist and if the testing is designed well.

4.3 'Rule of Thumb' Approximations / Relations

An easy way to get started with the design of an aircraft is by using simple relations found on hobbyist sites [13] or given by experienced builders. Knowing what these refer to and how they benefit the craft is more important though. This knowledge provides a designer the ability to root out incorrect/unhelpful relations, and provide explicit reasoning behind every dimensional consideration. These are commonly used rules of thumb from a hobbyist website for analysis [13].

- "ratio of the wing span to wing root chord should be 5 or 6"

  This refers to the aspect ratio of a plane. Usually the larger the aspect ratio, the more stability a plane has, and conversely the lower the aspect ratio, the more controllable it is. In part, this is due to the increased moment of inertia of a higher aspect ratio wing, as inertia scales with the square of the radius its turning about. That means a larger disturbance is necessary to cause the same variance from equilibrium on a plane with a larger aspect ratio. The Aero 2020 plane 3 has an aspect ratio of 7.5, though aspect ratio often depends more on the use of the plane, and surface area requirements of the wing should be considered first.

- "The wing thickness should be 12% to 14% of the wing root chord"

  This refers to the normal shape of an airfoil, for example as mentioned previously the wing airfoil of the Aero 2020 plane is 15.7%, and the tail 12%. Using an airfoil will almost guarantee this, but making planes from cut foam or poster board this is a good approximation. Use an airfoil instead, if there are strict design constraints.

- "The aileron surface area should be 10% - 12% of half of the wing surface"

  When comparing to Aero 2020, the textbook-based aileron sizing algorithm produced ailerons in total 12% the surface area of the whole wing, so this range provided is reasonable, if a little low for heavy planes.

- "The fuselage length should be 70% - 75% of the wing span"

  This seems arbitrary, and really depends on the function of the plane. If it must carry things, the fuselage potentially must be larger than this. If it doesn't have to carry anything, there's no need for a fuselage on a plane. This can potentially waste time, physical and capital resources, and effort if not strictly necessary. Fuselages change the dynamics of the plane greatly at higher angles, adding flat surfaces to the craft for lift and drag to act.

- "Distance from the wing's leading edge to back of the prop should be 15% of the wingspan"

  This forces the motor to be farther in front of the wing, moving the CG farther forwards and closer to a viable position.

- "The leading edge of the wing to the stabilizer should be 3 times the wing root chord"

  This forces the tail moment arm to be a certain length relative the wing. If anything, it should be farther back, but the closer the tail is to the wing the larger surface areas the tail should contain. For example, if the Aero 2020 plane had this short a tail, 16.5*3 inches back, the stability calculation now rates the plane the same as if the tail area were lowered.

- "The horizontal stabilizer should be 25% of the wing area"

  The Aero 2020 plane's horizontal stabilizer is 25% exactly, and although other relations for tail design dictate it should be smaller, flight testing and static analysis with the code above show 25% or more is desirable.

- "The elevator (attached to the HS) should be 25% of horizontal stabilizer surface area"

  Fixing the initial size of the elevator guarantees a base level of control, although control depends on stability, and stability depends on the purpose of the plane, dimensions and contents. Very often more is needed, but this statistic should be supported with analysis. This requires more analysis, though is not a bad approximation.

- "The vertical stabilizer should be 10% of the wing area"

  This provides a base level of roll/yaw stability, though again depending on the different uses of the plane this is not guaranteed to be enough. This requires more analysis, though is not a bad approximation for initial analysis.

- "The rudder (attached to the vertical stabilizer) should be 25% of the vertical stabilizer surface area"

  (Same evaluation as the elevator relation above)

- "The plane should balance at 25% - 33% of the wing root chord"

  This places the CG of the plane right at or just behind the AC, forcing the lift to have a positive moment to counteract the moment from the tail. Using the static analysis, placing the CG in front of the AC produces a higher correcting moment and increases the equilibrium angle slightly. All in all, this statistic really just makes a margin the works, though this is not the margin that works and there is no real reason to only keep the CH within those bounds. Anywhere from 0% to 50% can work fairly well depending on other plane dynamics and dimensions, though it may be easier to place the CG from 0-30% of the chord.

Rules of thumb can provide rough initial dimensions for a craft, as something to build on. its limitations comprise their accuracy and relevancy. If there is no knowledge about what a rule of thumb is based off of, then its possible that it is either completely arbitrary or the range of dimensions given are not a good place to start. All in all, most of these suggestions are helpful in that they set a value to something that didn't have one before and are not detrimental to the design, but also don't provide any reasoning or optimal circumstances for plane design. Static analysis along can provide important insight, but there is no one configuration that ultimately is best, and that is why crafts should be analysed individually and decisions made well informed.

4.4 Undesirable Stability

Stability sounds like something necessary to maximize, but stability is the antithesis of maneuverability. Light military crafts and other acrobatic planes need a precise and wide range of movement and so they have lower stability than, for example, commercial aircrafts and cargo planes. If a craft has heavy payload typically it has higher stability so that the payload, and the craft, is not lost. Lighter planes can correct faster because of lower mass moments of inertia among other things, so they can operate with a lower stability more safely.

Chapter 5

Conclusions and Recommendations

5.1 Design Improvement Decisions

Having a model for static analysis allows the ability to review crafts and calculate what could be changed to increase stability. The majority of Union College SAE Aero competition planes for the last several years have had similar configurations; planes have had conventional aft tails with top wings, and a motor in front that pulls it through the air, in-line with the center of gravity. The model generated by this composition pertains to this type of craft, and although this configuration of parts is common for a large selection of planes, there exist an uncountable number of combinations. Many different types of tails, wings, landing gear, and surfaces on a craft change or add forces and alter the moment response. The configuration of a plane and the geometry of its parts are first order variables in the stability of a craft. If a plane does not conform to the configuration/assumptions that section (2)'s analysis is based around, it can be modified to present the same data. Bottom line, if a plane deviates from the model assumptions, either alter or add terms to receive useful information.

Since this model pertains well to the Union College Aero planes, the planes can provide an example for analysis. Looking at the results in table 2, the Aero 2020 plane could be made more statically stable with a larger tail chord, and if there are large portions of the fuselage that contain flat surfaces, then an angle closer to zero may be a more stable equilibrium. Otherwise, a large amount of lift and drag caused by these surfaces at an angle may make the moment plot (figure 9) show a non-linear slope at lower angles of attack. The main takeaways from this analysis are that if more control is needed, alter dimensions to lower the magnitude of the moment derivative, and more stability requires a larger negative moment slope. The farther the angle from equilibrium, the less accurate this model will be at analysing the craft, and the more flat surfaces added onto the plane that are not lifting surfaces (the tail or wing), the more effects this model misses. In this case, do not throw away this model, but build on it.

5.2 Stability Regulations

A number of these analyses produce quantitative values associated with the craft, and can be compared to other crafts. This means commercial and private crafts can be characterized and regulated. However they're calculated, with a program, physical, or dynamic model, plane characterization usually classify each as a certain class of craft [14], and then values of control derivatives or turn timing or damping ratio can be constrained within a certain range.

As an example of this, some old military regulations concerning classes of planes and their respective ranges of control characterization values for plane design are presented in figure 11. This is also a very good resource if a stability characterizing value for a craft is not know, as this provides ranges usable for design. The original document is fairly verbose, though Sadraey's textbook summarises and separates planes in to classes of vehicle based on use [5]. Each has three phases of flight, and within that three levels of comfortability for the pilot. Then damping ratios, control derivatives, and (provided here) roll and pitch times are specified [14]. It is hard to intuitively predict values for turning times, and these are a useful way to design control surfaces in a way that reflects many crafts still flown today. Other regulations by the FAA and similar organizations can also be a source if there is no reference for what a stability characterizing value should be.

Class	Aircraft characteristics
I	Small, light aircraft (maximum take-off mass less than 6000 kg) with low maneuverability
II	Aircraft of medium weight and low-to-medium maneuverability (maximum take-off mass between 6000 and 30 000 kg)
III	Large, heavy, and low-to-medium maneuverability aircraft (maximum take-off mass more than 30 000 kg)
IV	Highly maneuverable aircraft, no weight limit (e.g., acrobatic, missile, and fighter)

Category	Examples of flight operation
A	(i) Air-to-air combat (CO); (ii) ground attack (GA); (iii) weapon delivery/launch (WD); (iv) aerial recovery (AR); (v) reconnaissance (RC); (vi) in-flight refueling (receiver) (RR); (vii) terrain following (TR); (viii) anti-submarine search (AS); (xi) close formation flying (FF); and (x) low-altitude parachute extraction system (LAPES) delivery.
B	(i) Climb (CL); (ii) cruise (CR); (iii) loiter (LO); (iv) in-flight refueling in which the aircraft acts as a tanker (RT); (v) descent (D); (vi) emergency descent (ED); (vii) emergency deceleration (DE); and (viii) aerial delivery (AD).
C	(i) Take-off (TO); (ii) catapult take-off (CT); (iii) powered approach (PA); (iv) wave-off/go-around (WO); and (v) landing (L).

No.	Aircraft type	Rotation time during take-off (s)	Take-off pitch angular acceleration (deg/s^2)
1	Highly maneuverable (e.g., acrobatic GA and fighter)	0.2–0.7	12–20
2	Utility, semi-acrobatic GA	1–2	10–15
3	Normal general aviation	1–3	8–10
4	Small transport	2–4	6–8
5	Large transport	3–5	4–6
6	Remote control, model	1–2	10–15

(a) Time to achieve a specified bank angle change for Class I

Level	Flight phase category		
	A	B	C
	Time to achieve a bank angle of 60° (s)	Time to achieve a bank angle of 45° (s)	Time to achieve a bank angle of 30° (s)
1	1.3	1.7	1.3
2	1.7	2.5	1.8
3	2.6	3.4	2.6

Figure 11: Original military guidelines are no longer used by the military though are still very comprehensive for design [14], as summarised for use in Sadraey's text [5] for that reason. (Sadraey, M., 2013, *Aircraft Design; a Systems Engineering Approach,* Wiley, United Kingdom.)

5.3 Personalized Analysis Recommendations

The best way to get an analysis to describe any craft is to design it based around that craft. This composition focuses on presenting much of the surrounding information on plane stability to ease the construction of static models, provide the ability to make informed dimensional decisions, and recommended sources to go further. Dynamic stability requires more background for it to be done well, but with available online courses, documented computational programs, and good physical testing, a student or small business has the ability to complete a comprehensive analysis. After understanding the baseline principles and the uses of different analyses within this text, a list of possible sources is presented below to provide direction for further research.

- "Aircraft Stability and Control," Massachusetts Institute of Technology, [2]
- Introduction to Aircraft Stability and Control, Cornell University, [7]

Both of these are courses in aeronautic by respected institutions. They give an explanation of both the static stability content covered here and dynamic stability beyond, though in more complex terms. The one problem is that this is a full upper level class, so it builds off itself and it is difficult to refer to one part without much of what comes before.

- Sadraey's *Aircraft Design; a Systems Engineering Approach,* [5]

This textbook gives a very useful explanation of the design order of an aircraft from scratch. For small craft design there may be disregardable sections, for example including seats in the plane for passengers, but overall it is very useful and old enough that it is available for free online.

- http://airfoiltools.com/ [11]

If an airfoil is ever needed, chances are the Airfoiltools database has a suitable design that has been thoroughly tested already. This is an easy to use database of thousands of airfoils.

- "A Comparison of CFD Modeling to Other Numerical Methods for the Union College Aero Team," Mechanical Engineering Thesis, Union College. [10]

This is the senior thesis of Keving Caro, [I do have his permission], hitting on a lot more of CFD and involved computational methods used to estimate lift and drag over a wing.

- "A Procedure for Estimating Stability and Control Parameters from Flight Test Data by Using Maximum Likelihood Methods Employing a Real-Time Digital System," National Aeronautics and Space Administration, [15]

If flights are possible, this details how to use the flight test information to its maximum potential. Wasting a flight test is a large waste of resources, so this can help mitigate the waste.

- "Refined Phugoid Approximations for Conventional Aircraft," American Institute of Aeronautics and Astronautics, [16] This details a more accurate method of producing the phugoid, showing high accuracy in natural frequency and damping ratio. This method is then paired to a large database of flight conditions to test its accuracy. If phugoids can be developed, it would be useful to explore either the accuracy modification or the database of values to compare to.

Stability and its characterizing values are fairly important to the design of a functional plane. This composition only scratches the surface and provides a good idea of the parameters involved in stability and different methods of quantifying it, in the hope to ease the learning of this information. The 'barrier of entry' that this information creates is one that must be overcome for detailed analysis, being the method that involves the least amount of physical and capital resources.

References

[1] Cengel, Y. and Cimbala, J., 2014, *Fluid Mechanics Fundamentals and Applications,* 3rd ed., McGraw Hill, New York, NY.

[2] How, J., 2004, "Aircraft Stability and Control," Massachusetts Institute of Technology, from https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-333-aircraft-stability-and-control-fall-2004/index.htm.

[3] Mascolo, I., 2019, “Recent Developments in the Dynamic Stability of Elastic Structures,” Frontiers in Applied Mathematics and Statistics, 5(1), pp. 1-10.

[4] Ogata, K., 2010, *Modern Control Engineering,* 5th ed., Prentice-Hall, India.

[5] Sadraey, M., 2013, *Aircraft Design; a Systems Engineering Approach,* Wiley, United Kingdom.

[6] Bruno, A. and Caro, K. and Khan, M. and Landreth, R. and Santos, Joana, A., 2020, “Union College Flying Dutchmen Advances Design Report,” Union College, Schenectady NY, p. 27.

[7] Caughey, D, 2011, Introduction to Aircraft Stability and Control, Cornell University, p 94 from https://courses.cit.cornell.edu/mae5070/Caughey_2011_04.pdf.

[8] Irving, F., 1966, *An Introduction to the Longitudinal Static Stability of Low-Speed Aircraft,* Pergamon Press, Oxford.

[9] Rosario-Gabriel, I., and H. Rodriguez Cortes, 2018, “Aircraft Longitudinal Control Based on the Lanchester’s Phugoid Dynamics Model,” 2018 International Conference on Unmanned Aircraft Systems (ICUAS), IEEE, pp. 924–29.

[10] Kevin Caro, 2020, "A Comparison of CFD Modeling to Other Numerical Methods for the Union College Aero Team," Mechanical Engineering Thesis, Union College.

[11] "Douglas LA-203a Airfoil," http://airfoiltools.com/airfoil/details?airfoil=la203a-il.

[12] "E168 (12.45%) (e168-il)," http://airfoiltools.com/airfoil/details?airfoil=la203a-il.

[13] Homebuilt Airplanes, 2013, "’Rule of the Thumb’ to designing a plane...," from https://www.homebuiltairplanes.com/forums/threads/rule-of-the-thumb-to-designing-a-plane.15668/.

[14] 1980, "MIL-F-8785C, Military Specification: Flying Qualities of Piloted Airplanes," US Military, from http://everyspec.com/MIL-SPECS/MIL-SPECS-MIL-F/MIL-F-8785C_5295/.

[15] Grove, R., Bowles, R. and Maybew, S., 1972, "A Procedure for Estimating Stability and Control Parameters from Flight Test Data by Using Maximum Likelihood Methods Employing a Real-Time Digital System," National Aeronautics and Space Administration, Hampton, Va.

[16] Kamesh, S and Pradeep, S., 1998, "Refined Phugoid Approximations for Conventional Aircraft," American Institute of Aeronautics and Astronautics, Inc, Boston, MA.

Appendix A

Longitudinal Static Stability Script

```
% Roderick Landreth
% Aero Advanced 2020 Senior Thesis
% Longitudinal Static Stability Calculation and Visualization
clear

wing_span = 123.25;                     % inches
wing_chord = 16.5;                      % inches
wing_thickness = wing_chord * 0.157;    % inches
distance_cg_to_ac = 5.5-4.125;          % inches
tail_span = 55;                         % inches
tail_chord = 9.22;                      % inches
tail_thickness = tail_chord * 0.1245;   % inches
wing_ac_to_tail_ac = 70.1;              % inches
wing_angle_of_incidence = 3;            % degrees
tail_angle_of_incidence = 0;            % degrees
motor_angle_of_incidence = 0;           % degrees
static_thrust = 20*30;                  % lbs *in (this doesn't
                                            %matter if angle is zero)

z = Longitudinal_Stability(wing_span,wing_chord,wing_thickness,...
    distance_cg_to_ac,tail_span,tail_chord,tail_thickness,...
    wing_ac_to_tail_ac,wing_angle_of_incidence,...
    tail_angle_of_incidence,motor_angle_of_incidence,static_thrust);

function cl = Wing_Lift_Coeff(ang)
    cl = 0.7;
    if ang < -10.0
        cl = 0.0;
    elseif -8.0 <= ang && ang <= 10.0
        cl = 0.000669839*ang^2 + 0.104523*ang + 0.699226;
    elseif 10.0 < ang && ang <= 11.75
        cl = 1.7;
    elseif 11.75 < ang && ang <= 17.0
        cl = 2.17515 - 0.0396471*ang;
    end
end

function cl = Tail_Lift_Coeff(ang)
```

```
    if ang > -12.5 && ang < 12.5
        cl = ang*0.1;
    elseif ang <= -12.5
        cl = -1.1;
    elseif ang >= 12.5
        cl = 1.1;
    end
end

function cd = Wing_Drag_Coeff(ang)
cd = 0.01282*ang - 0.1098;
    if ang < -2.0
        cd = 0.000140654 - 0.0103897*ang;
    elseif -2.0 <= ang && ang < 10.0
        cd = 0.0193;
    end

end

function cd = Tail_Drag_Coeff(ang)
    if ang <= -7.5
       cd = -0.014*ang -0.095 ;
    elseif ang >= +7.5
       cd = 0.014*ang - 0.095;
    elseif ang < 7.5 && ang > -7.5
        cd = 0.01;
    end
end

function cm = Wing_Moment_Coeff(ang)
    cm = 0;
    if ang <= 0 && ang > -10
        cm = ang*-0.009 - 0.18;
    elseif ang < 7.5 && ang >= 0
        cm = -0.18;
    elseif ang >= 7.5
        cm = ang*0.007 - 0.232;
    end
end

function momnt = moment(rho,v,th,xac,lt,bw,cw,tw,bt,ct,tt,a1,a2,a3)
momnt = 0.5 * rho * v^2 *( xac * bw * ( cw*...
        Wing_Lift_Coeff( a1 )*cosd( a1 ) - tw*...
        Wing_Drag_Coeff( a1 )*sind( a1 ) ) - lt*bt*...
        (ct*Tail_Lift_Coeff( a2 )*cosd( a2 )...
```

```
        - tt*Tail_Drag_Coeff( a2 )*sind( a2 )) - Wing_Moment_Coeff(...
         a1 )*bw*cw^2 ) + th*sind( a3 );
end

function [Cm0,CmPlus,CmMinus,dMdAlpha,angle_of_stability] = ...
    Longitudinal_Stability(bw,cw,tw,xac,bt,ct,tt,lt,aw,at,ath,th)
    v_defult = 25*17.6; %25mph to inches per second
    rho_defult = 0.000043256; %lbf/in^3
    da = 8;
    Cm0 = moment(rho_defult,v_defult,th,xac,lt,bw,cw,tw,bt,ct,tt,aw,at,ath)

    CmPlus = moment(rho_defult,v_defult,th,xac,lt,bw,cw,tw,bt,ct,tt,aw+da,...
            at+da,ath)

    CmMinus = moment(rho_defult,v_defult,th,xac,lt,bw,cw,tw,bt,ct,tt,aw-da,...
            at-da,ath)

    dMdAlpha = (CmPlus - CmMinus)/(2*da)
    angle_of_stability = -Cm0/dMdAlpha

    if abs(angle_of_stability) < 8 && dMdAlpha < 0
        describe = strcat('This plane is statically stable at %f degrees',...
            '. Positive static stability.');
        fprintf(describe,angle_of_stability)
    elseif abs(angle_of_stability) > 8 && dMdAlpha < 0
        describe2 = strcat('Fix some stuff. This plane would be statically',...
            'stable at %f degrees, though it would probably stall',...
            ' and crash before then. Neutral static stability.');
        fprintf(describe2,angle_of_stability)
    elseif dMdAlpha > 0
        describe2 = strcat('You gotta change a lot. This is negative',...
            ' static stability, with these assumptions it will not fly.');
        disp(describe2)
    end

    angleRange = (-10:0.25:10);
    N = length(angleRange);
    momentRange = zeros(N);
    %disp(length(angleRange))
    %disp(length(momentRange))
    for n = 1 : N
        totalWingAngle = aw + angleRange(n);
        totalTailAngle = at + angleRange(n);
       momentRange(n) = moment(rho_defult,v_defult,th,xac,lt,bw,cw,...
           tw,bt,ct,tt,totalWingAngle,totalTailAngle,ath);
```

```
    end
    figure
    hold
    ylabel('Moment about Center of Gravity (lbf-in)')
    xlabel('Craft angle of Attack (Degrees)')
    plot(angleRange,momentRange)
    plot(angle_of_stability,0,'o')
    text(angle_of_stability,0,'Stability Angle','VerticalAlignment',...
        'bottom','HorizontalAlignment','left')

end
```