THE DETERMINATION OF

PARTICLE TERMINAL AND TRANSPORT VELOCITIES

A thesis presented to the Department of Chemistry of Union College in partial fulfillment of the requirements for the degree of Master of Science.

By Bertram C. Raynes

Approved by Charles B. Hurd

Approved for the Committee on Graduate Studies by Leonard B. Clark

March 15, 1949

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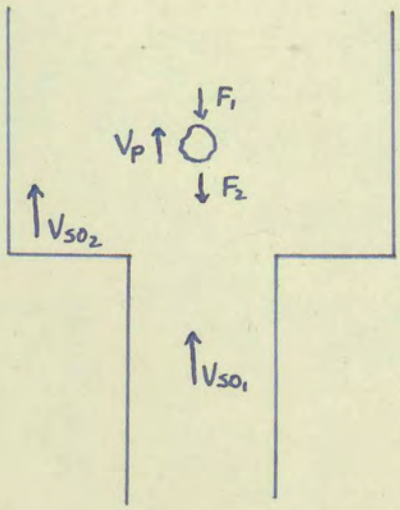
INTRODUCTION

The movement of finely divided material in chemical process equipment is becoming the object of much present-day study.(1) In industrial practice, the ability to predict the behavior of small bodies moving through a fluid medium or being moved by a fluid medium must be based upon data relating to physical properties of both the particle and the fluid; these physical properties should be those which can be readily determined.

A particle falling through a fluid medium will achieve a steady state velocity if it is allowed to fall until its acceleration becomes zero. This steady state rate of fall is defined as the particle terminal velocity. The velocity at which a particle of known terminal velocity will be carried by a fluid medium is defined as particle transport velocity. It is convenient to consider the velocity of the fluid medium to be based upon volume rate of flow, and the geometrical area of its conduit; this velocity is the superficial velocity. The present study limits itself to the behavior of particles falling in still air. It will be shown that particle terminal velocities can be predicted from simple physical properties. These terminal velocities will be suitable for industrial chemical calculations. From the knowledge of terminal velocities, transport velocities and settling rates will be calculated.

THEORY

Consider a particle being carried vertically upwards in a conduit by a gaseous fluid whose superficial velocity is V_{so_1}. This particle enters a larger conduit in which the superficial velocity has diminished, V_{so_2} . Let F_1 be the force acting downwards on the particle due to viscous drag.* Let F_2 be the force acting downwards due to the weight of the particle. Finally, let V_p be the particle velocity.



$$F_1 = K_1 \mu_f D_p (V_p - V_{so_2})$$

$$F_2 = m_p g$$

* A complete notation of symbols used will be found on Page 20 of this thesis.

The total force acting downward is the sum of F_1 and F_2. The rate of change of particle velocity, $\frac{dV_p}{dt}$, must be equal to the total force acting divided by the mass of the particle. The mass of the particle, m_p is $= K_2 D_p^3 \rho_p$ where K_2 is a volume factor, ρ_p is the mass density, and D_p is particle diameter. Therefore, we may write:

$$(1) \quad -\frac{dV_p}{dt} = \frac{K_1 \mu_f D_p (V_p - V_{SO_2}) + K_2 D_p^3 \rho_p g}{K_2 D_p^3 \rho_p}$$

Rearranging

$$(2) \quad -\frac{K_2 D_p^3 \rho_p}{K_1 \mu_f D_p} \int \frac{dV_p}{\left[(V_p - V_{SO_2}) + \frac{K_2 D_p^3 \rho_p g}{K_1 \mu_f D_p}\right]} = \int dt$$

Integrating

$$(3) \quad -\frac{K_2 D_p^3 \rho_p}{K_1 \mu_f D_p} \log \left[V_p + \frac{K_2 D_p^3 \rho_p g}{K_1 \mu_f D_p} - V_{SO_2}\right] = t + c$$

Or

$$(4) \quad V_p + \frac{K_2 D_p^3 \rho_p g}{K_1 \mu_f D_p} - V_{SO_2} = e^{-\frac{K_1 \mu_f D_p}{K_2 D_p^3 \rho_p}[t+c]}$$

If we now consider the particle to be falling at its terminal velocity, V_o, with V_{so} taken as zero, then there is no acceleration and the resistance offered to the fall of the particle is due to the viscous drag or F_1.

F_1 is found in the following manner. It has been shown that the resistance to particle fall depends upon the viscosity of the fluid medium, μ_f , the particle velocity V_o, and the particle diameter, D_p. Expressed in terms of their physical dimensions:

Viscosity, $\mu_f = [ML^{-1}T^{-1}]$

Velocity, $V_o = [LT^{-1}]$

and Diameter, $D_p = [L]$

A force, F, must have the dimensions $[MLT^{-2}]$.

Since $F = k\,\mu_f^x V_o^y D_p^z$,

Then $k = \dfrac{F}{\mu_f^x V_o^y D_p^z}$. And

$$[k] = \frac{[MLT^{-2}]}{[ML^{-1}T^{-1}]^x[LT^{-1}]^y[L]^z}$$

Or $[k] = M^{1-x}\,L^{1+x-y-z}\,T^{-2+x+y}$

But k must be dimensionless, so x, y and z must all equal 1.

Therefore

$$F_1 = k\,\mu_f V_o D_p.$$

At steady state conditions the resistance to fall equals the downward gravitational force.

A. $K_1 \mu_f V_o D_p = K_2 D_p^3 \rho_p g$

Rearranged, B. $V_o = \frac{K_2 D_p^3 \rho_p g}{K_1 \mu_f D_p}$

Substituting the value of $\frac{K_2 D_p^3 \rho_p g}{K_1 \mu_f D_p}$ in Equation (4)

(5) $V_p = V_{SO_2} - V_o + e^{-g/V_o[t+c]}$

The constant C in Equation (5) depends upon the boundary conditions, and has a small finite value. At the steady state conditions imposed upon this consideration, time t in Equation (5) is infinite and the final term vanishes.

The equation then says that particle velocity is equal to the superficial gas velocity minus the particle terminal velocity.

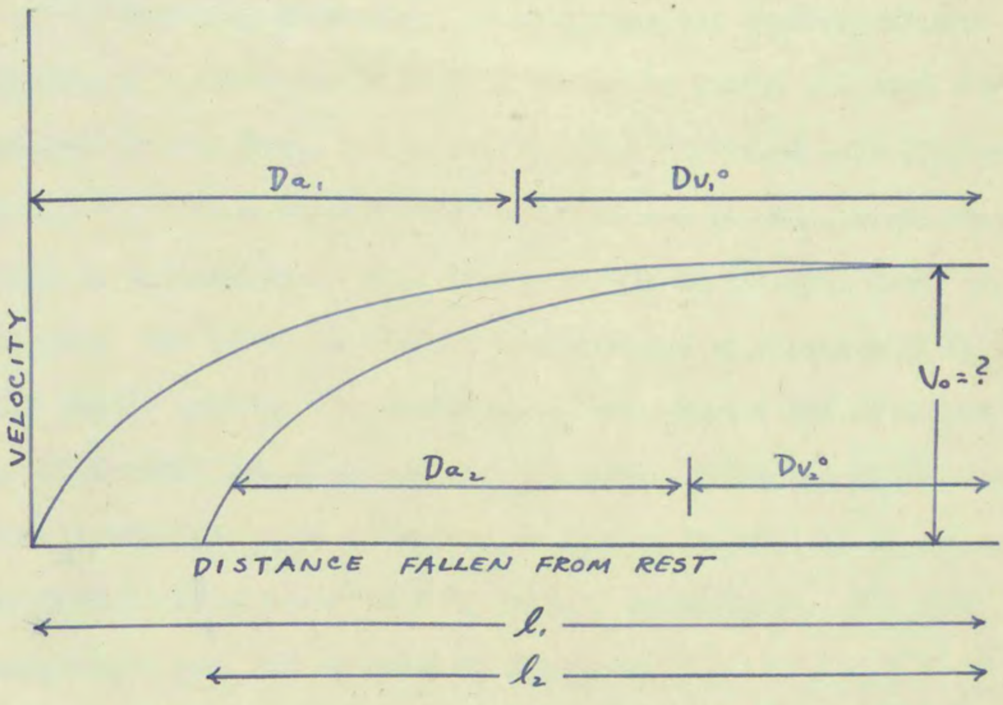
(6) $V_p = V_{SO_2} - V_o$

Thus if the terminal velocity of the particle is known and if the superficial velocity of the fluid can be calculated, the actual particle velocity or transport velocity can easily be determined. This entire discussion is based on the assumption that the particle is unaffected by other particles falling in the conduit and that the conduit is large enough to obviate frictional and wall effects.

GENERAL CONSIDERATIONS AND EXPERIMENTAL PROCEDURE

Many studies have been reported describing settling rates of particles in liquids. Few have been made in a gaseous medium because of the experimental difficulties. In this study, the problem was approached from the following point of view.

If particle velocity is plotted against distance fallen for two different distances, the graph looks like this:



D_a = distance of acceleration; at $V_o, \frac{dv}{dt} = o.$

Dv^o = distance of constant terminal velocity fall.

The assumptions are made here that the same fluid medium at constant conditions is employed and that the distance of acceleration, D_a, is constant; i.e. $D_{a_1} = D_{a_2}$.

Then (1) $Dv_1^\circ - Dv_2^\circ = l_1 - l_2$

And (2) $V_o t_1 - V_o t_2 = l_1 - l_2$

Therefore (3) $V_o = \frac{l_1 - l_2}{t_1 - t_2}$

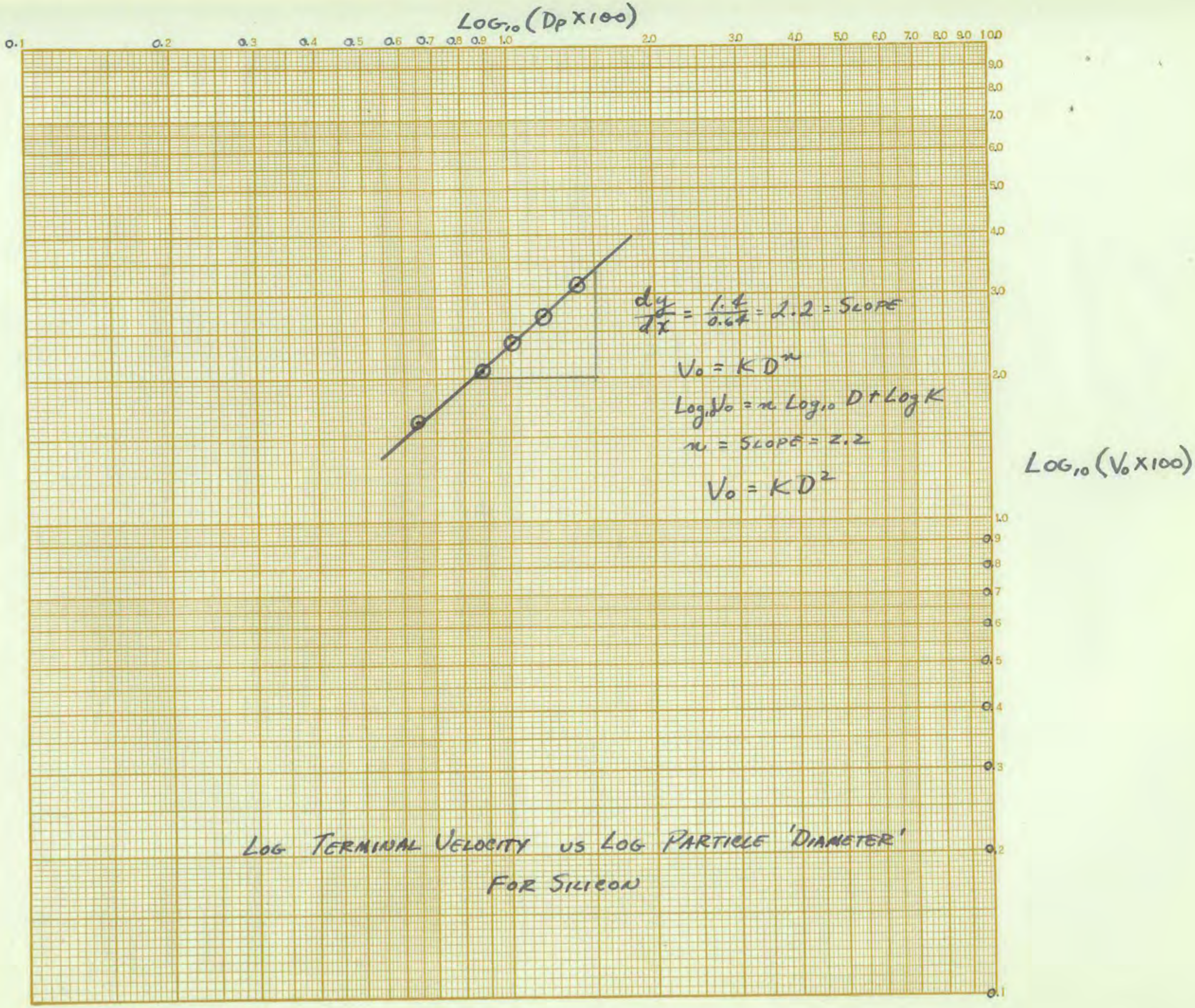
To test this conclusion, an apparatus was constructed consisting of a glass tube 2.25 ± 3% inches in inside diameter and exactly 12 feet long, set up vertically. Particles were dropped at the top from an electrically operated cup so constructed that a lip is released and swings downward when an electric timer is started. The timer and the cup are operated simultaneously by a push button switch. The experiments were carried out in a room in which there was no detectable movement. There was no air movement through the glass tube, due to thermal reasons or any other, which could be measured on a laboratory annemometer. The room temperature was held essentially constant.

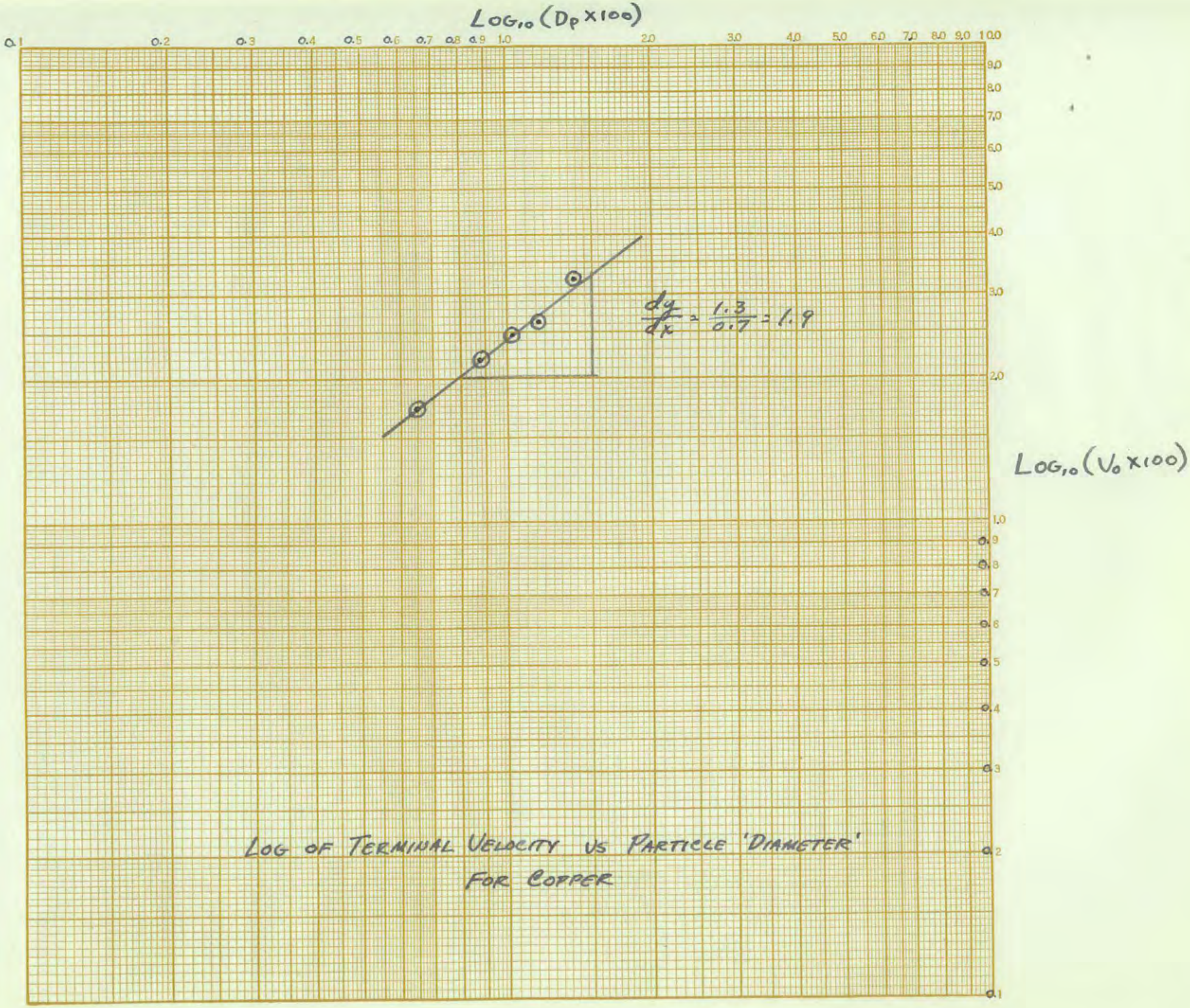
The timer was stopped when particles were seen to arrive at one of the three levels, six feet, nine feet, and twelve feet. A small wooden cylinder whose distance from the top could be changed readily provided these levels. Since the results of these experiments were intended for engineering usage, it was felt that visual observations would be adequate. As a check, the gravitational constant was determined using a ¼" glass ball. An error of only 0.3% was found.

EXPERIMENTAL RESULTS

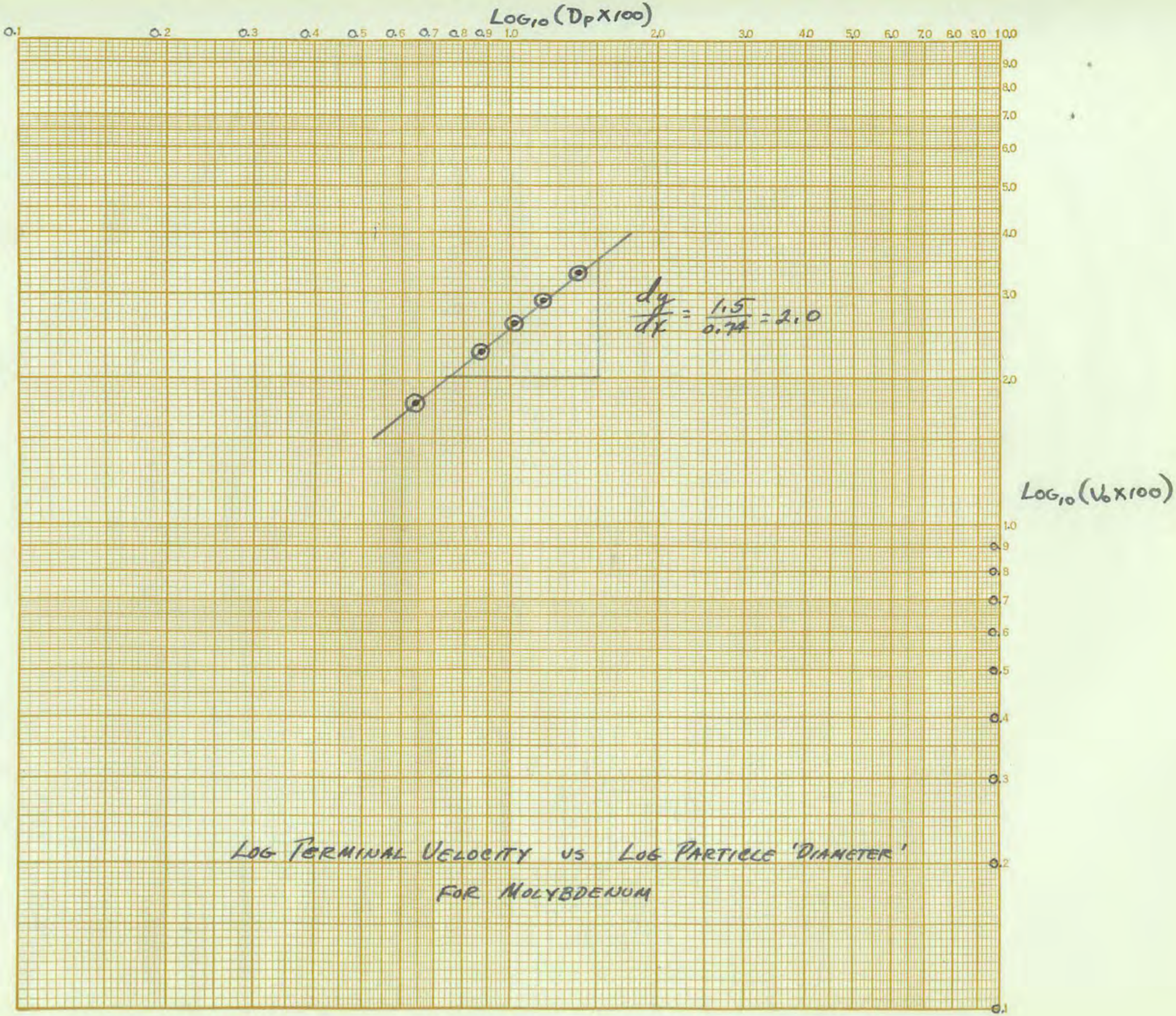
The observed particle terminal velocities are presented in Table 1. The velocities represent averages of at least five separate observations and, in many cases, as high as twelve observations. Particles of silicon, copper, molybdenum, and tungsten were studied. The density range covered by these elements is some eight-fold. The size of the particles ranged from 40 microns to about 250 microns in diameter.

Plotted on logarithmic graph paper, the data show that the terminal velocities attained by these particles is proportional to the square of their diameters, in agreement with the general equation $V_o = KD^2$. Further examination of the data reveals that the constant K must equal $\frac{k}{\mu_f}\left(\frac{\rho_p}{\rho_p+1}\right)$. The terminal velocity of a particle is found to be directly proportional to the square of its diameter, inversely proportional to the viscosity of the fluid medium through which it is falling, and proportional to the ratio of its specific gravity over its specific gravity plus unity. The proportionality constant, k, is found to be numerically equal to 360.

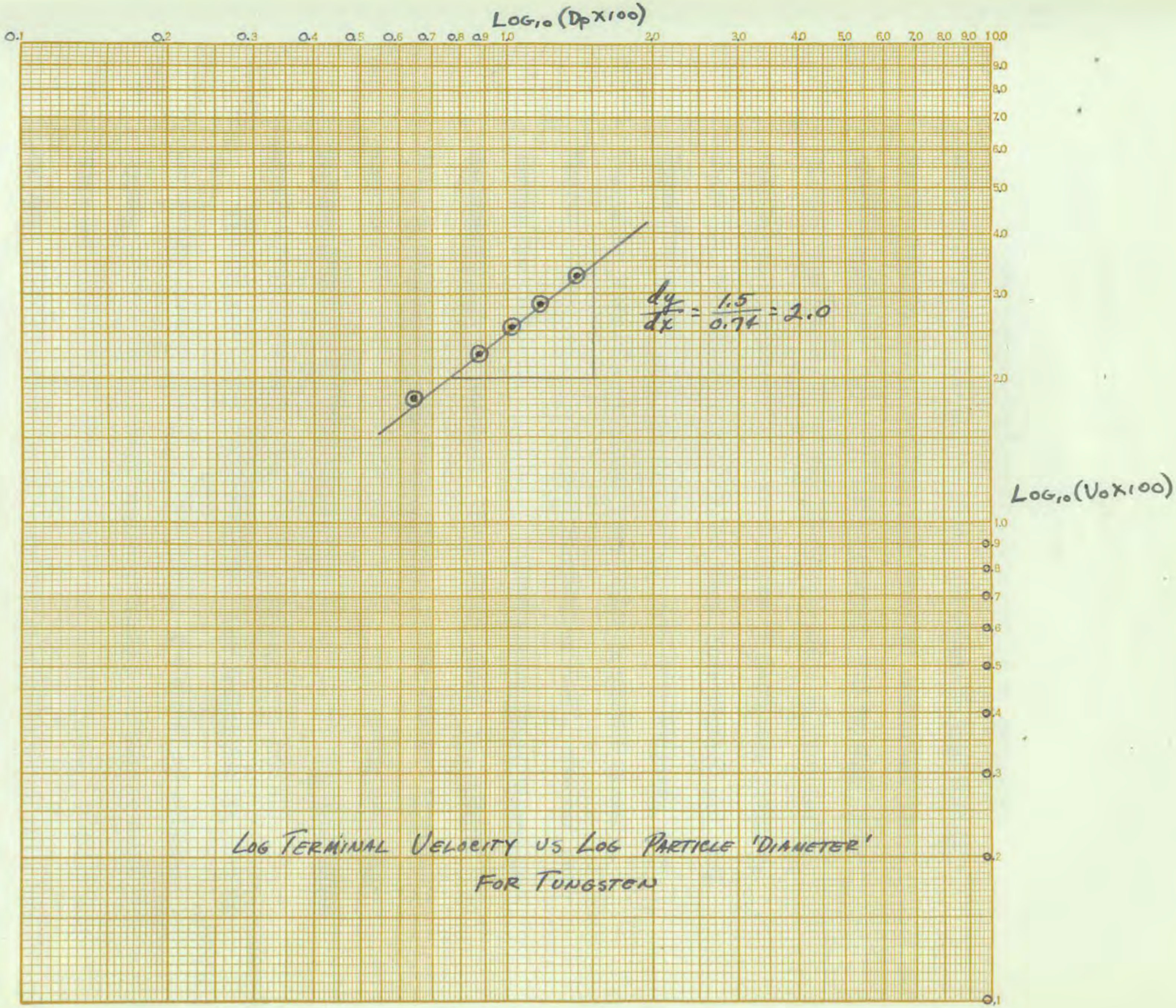

$Log_{10}(D_p \times 100)$
$Log_{10}(V_0 \times 100)$
$\frac{dy}{dx} = \frac{1.4}{0.64} = 2.2 = SLOPE$
$V_0 = KD^n$
$Log_{10} V_0 = n \, Log_{10} D + Log K$
$n = SLOPE = 2.2$
$V_0 = KD^2$
LOG TERMINAL VELOCITY VS LOG PARTICLE 'DIAMETER'
FOR SILICON



$\text{Log}_{10}(D_p \times 100)$
$\text{Log}_{10}(V_0 \times 100)$
$\frac{dy}{dx} = \frac{1.3}{0.7} = 1.9$
LOG OF TERMINAL VELOCITY VS PARTICLE 'DIAMETER'
FOR COPPER



Log10 (Dp X 100)
Log10 (Vo X 100)
dy/dx = 1.5/0.74 = 2.0
Log Terminal Velocity vs Log Particle 'Diameter'
For Molybdenum



LOG10 (Dp X100)
LOG10 (Vo X100)
dy/dx = 1.5/0.74 = 2.0
LOG TERMINAL VELOCITY VS LOG PARTICLE 'DIAMETER'
FOR TUNGSTEN

DISCUSSION

In many fields powdered or crystalline or granular materials are of primary concern. In chemical engineering, in the science of geology, in the study of hydrology, in mineral physics, in the ceramic industry, and in many other fields, the knowledge of the characteristics of fine particles is of importance[2]. The problem of particles settling under the influence of gravity has been investigated and different laws expressing their behavior have been derived. For the upper ranges of particle size, above about 100 microns, Stokes' law is said to express the rate of fall. Stokes' law states that the rate of fall of a particle is proportional to the square of its diameter. Above these sizes, the rate of fall is stated to be directly related to the density difference between the particle and the fluid. Very small particles diffuse almost like a gas and settle according to the Cunningham modification of Stokes' law.

The present study recognizes that the resistance to fall is due to the viscous nature of the fluid. Particles of silicon, copper, molybdenum and tungsten were studied falling in still air. Under the microscope these particles are found to have essentially the same particle shape so that corrections of rates of fall due to resistances offered to varying geometrical shapes may be neglected.

These particles were found to fall according to the following law: $V_o = \frac{k}{\mu_f}\left(\frac{\rho_p}{\rho_{p+1}}\right) D^2$. K is found to be 360.

By the use of this equation rates of fall may be estimated for particles falling in any gaseous medium. Vo can be calculated by use of the terminal velocity law. The superficial velocity, Vso, can readily be calculated from volume rates of flow. Then substitution of these values in Equation 6, Vp=Vso-Vo, will yield the particle velocity, Vp. Equation (5) can be used to find the height and cross sectional area which a settling chamber must possess in order to allow particle settling to take place. This is an application of increasing importance in industry. A correctly designed settling chamber would be the right height and length just to catch the desired particles and to prevent them from leaving the vessel.

As an example, Equation (5) will be used to find the height of chamber necessary to settle out a particle whose terminal velocity is 1 ft/sec. This particle has been carried in a pipe in which the superficial gas velocity was 7 ft/sec and it enters a settling chamber in which the gas velocity is 0.2 ft/sec.

$V_o = 1$ ft/sec $\quad V_{so_1} = 7$ ft/sec $\quad V_{so_2} = 0.2$ ft/sec

$$(5) \quad V_p = V_{so_2} - V_o + e^{-g/V_o[t+c]}$$

At the instant before the particle enters the settling chamber the particle is at steady state, $t = \infty$, and the last term is zero. Vp is then 6 ft/sec. At the instant the particle enters the settling zone, the boundary conditions are $t = 0$, Vp = 6 ft/sec. and Vso 0.2 ft/sec.

Substituting in (5) and solving for the constant c, we then have

$$V_p = 0.2 - 1 + e^{-g\left[t - \frac{\log 6.8}{g}\right]}$$

If the particle is to settle, Vp must pass through zero and

$$0 = 0.2 - 1 + e^{-g\left[t - \frac{\log 6.8}{g}\right]}$$

Solving for t, <u>t = 0.066 second</u>.

To solve for the distance required, differentiate Equation (5) with respect to time, t.

$$\frac{dV_p}{dt} = a = 0 - 0 - g/V_o\, e^{-g/V_o[t+c]}\, dt$$

From the expression for acceleration $s = \frac{1}{2} a t^2$, we know

$$\frac{ds}{dt} = a t\, dt$$

Substituting $ds = -g/V_o\, e^{-g/V_o[t+c]}\, t\, dt$

Solving, with time increasing from 0 to 0.066 second, we find $S = 0.31$ feet. Thus the settling chamber should have an area thirty-five times that of the conduit and should be (with engineering safety factors) at least one foot in length.

Equation (5) can therefore be used to solve problems involving particle settling when particle terminal velocities are known and if the change in superficial gas velocity is not so great as to show impact effects upon the particle.

It is interesting to compare the effectiveness of settling by the settling chamber method versus centrifugal powder removal such as is found in cyclone traps or in baffled sections. The velocity attained by a particle in a cyclone is given in the expression

(I) $V_c = \frac{k d_o \mu}{S_o \rho D^2}$ where V_c = uniform velocity in the circular path

S_o = angular distance of gas

d_o = radial diameter the particle must move out of the bas stream

D = particle diameter

Then (II) $d_o = \frac{V_c S_o \rho D^2}{k\mu} = K_o' V_{so}$, where $K_o' = \frac{\rho D^2}{k\mu}$.

Stokes' law in settling is

(III) $V = \frac{g d^2 \rho}{k\mu}$

It is apparent that

(IV) $d = \frac{gL}{V} = \frac{g L k\mu}{g d^2 \rho}$, where L = length of settling chamber

and $d_o = K_1' \frac{gL}{V}$, where $K_1' = K_o'$.

Substituting various velocities in these relations we can summarize as follows:

Conditions Imposed	Centrifugal Settling	Settling Chambers
Chamber 10 feet long	$d_o = K' V_e S_o$	$d_o = K' \frac{gL}{V}$
In baffled section, gas goes half circle Velocity is same as in settling section 1 ft/sec	$d_o = 3K'$	$d_o = 322K'$
Velocity is 5 ft/sec	$d_o = 15K'$	$d_o = 64K'$
Velocity is 10 ft/sec	$d_o = 30K'$	$d_o = 32K'$
Velocity is 15 ft/sec	$d_o = 45K'$	$d_o = 21K'$
Velocity is 20 ft/sec	$d_o = 60K'$	$d_o = 16K'$

It can be said, therefore, that if the velocity of the gas through a settling chamber is below the value of 10 feet per second (where Stokes' law is followed by the particles in falling) settling is more effective than centrifugal separation. Above 10 feet per second velocities, the centrifugal action will cause particles to move farther out of the gas stream and be more efficiently removed. For many applications gas velocities of less than 10 feet per second or less in a settling chamber are readily achieved, particularly in processes in which the volume rate of flow is relatively low. In these cases powder removal from the gas stream can be accomplished readily by means of a properly designed settling chamber.

TABLE NO. 1

Description	Mesh Size	Distance fallen feet	Time in Seconds Min Av.	Max Av.
Silicon	-60/100	6.0'	1.21	2.02
		9.0'	1.46	2.62
		12.0'	1.68	3.23
	-100/140	6.0'	2.00	3.61
		9.0'	2.67	4.84
		12.0'	3.32	6.20
	-140/200	6.0'	3.38	6.73
		9.0'	4.49	8.92
		12.0'	6.95	10.84
	-200/325	6.0'	6.98	15.57
		9.0'	9.40	22.54
		12.0'	12.37	30.04
Copper	-60/100	6.0'	0.99	1.63
		9.0'	1.17	2.19
		12.0'	1.35	2.62
	-100/140	6.0'	1.60	2.84
		9.0'	2.07	3.86
		12.0'	2.47	4.79
	-140/200	6.0'	2.76	5.08
		9.0'	3.75	6.92
		12.0'	4.78	8.08
	-200/325	6.0'	5.31	14.67
		9.0'	7.24	20.13
		12.0'	8.93	24.90

Table No. 1 is Continued

Table No. 1 Continued

Description	Mesh Size	Distance fallen feet	Time in Seconds Min Av.	Max Av.
Molybdenum	-60/100	6.0'	0.91	1.45
		9.0'	1.08	1.92
		12.0'	1.27	2.41
	-100/140	6.0'	1.46	2.67
		9.0'	1.89	3.54
		12.0'	2.35	4.37
	-140/200	6.0'	2.60	4.69
		9.0'	3.48	6.37
		12.0'	4.26	8.21
	-200/325	6.0'	5.11	12.31
		9.0'	6.74	18.97
		12.0'	9.80	24.15
Tungsten	-60/100	6.0'	0.83	1.23
		9.0'	1.00	1.68
		12.0'	1.16	2.10
	-100/140	6.0'	1.19	1.91
		9.0'	1.61	2.87
		12.0'	2.04	3.69
	-140/200	6.0'	1.88	4.04
		9.0'	2.86	5.49
		12.0'	3.36	7.02
	-200/325	6.0'	3.96	12.62
		9.0'	5.37	17.24
		12.0'	7.19	21.04

From the data in Table No. 1, velocities of particle fall are calculated in feet per second each measuring level. If these velocities agree within the experimental error, then the period of acceleration has been passed and the velocities represent terminal velocities. The results of these calculations are presented in Table No. 2

TABLE NO. 2

Description	Distance fallen feet	Size Particles	Velocities in ft/sec For Mesh Sizes 60	100	140	200	325
Silicon	6.0'- 9.0'	Max	12.8	5.00	2.44	1.35	0.43
	9.0'-12.0'	Max	13.7	4.93	2.20	1.56	0.40
	6.0'- 9.0'	Min		4.48	2.69	1.24	
	9.0'-12.0'	Min		5.22	2.35	1.01	
Copper	6.0'- 9.0'	Max	16.9	6.4	2.93	1.63	0.55
	9.0'-12.0'	Max	16.5	7.5	3.24	1.39	0.63
	6.0'- 9.0'	Min		5.36	3.02	1.56	
	9.0'-12.0'	Min		6.98	2.89	1.78	
Molybdenum	6.0'- 9.0'	Max	17.8	7.05	3.47	1.79	0.53
	9.0'-12.0'	Max	15.8	6.54	3.61	1.63	0.58
	6.0'- 9.0'	Min		6.52	3.39	1.84	
	9.0'-12.0'	Min		6.13	3.82	1.46	
Tungsten	6.0'- 9.0'	Max	17.4	7.20	3.11	2.07	0.65
	9.0'-12.0'	Max	18.2	6.93	3.65	1.96	0.79
	6.0'- 9.0'	Min		6.66	3.05	2.13	
	9.0'-12.0'	Min		7.15	6.00	1.65	

CONCLUSIONS

Particle terminal velocities have been determined for particles falling in a motionless gas (air). For particles which will pass a 60 mesh U. S. Standard sieve and for those which will just pass a 325 mesh sieve (particle "diameters" in the range 44 microns to 250 microns) it is possible to predict terminal velocities in a motionless gaseous fluid. The terminal velocities follow the law $V_o = \frac{k}{\mu_f}\left(\frac{\rho_p}{\rho_p + r}\right) D^2$, where μ_f is viscosity in centipoises, ρ_p is specific gravity of the solid, D is the particle diameter as determined from its mesh size. The factor, K, has the numerical value 360.

Particle terminal velocities can be used to calculate transport velocities where fluid superficial velocity is known. Particle terminal velocities can also be used in the design of settling chambers; a sample calculation of this nature is presented.

NOTATION

C - boundary constant in expression for transport and settling velocities

D_a - distance of particle acceleration

D_p - diameter of particle

D_{vo} - distance of the constant terminal velocity fall

E - resistance offered to particle fall in a gas; equals

F_1 - force acting downward upon a particle moving upward in a gas stream due to viscous drag

F_2 - force acting downward upon a particle moving upward in a gas stream due to its weight

g - gravitational constant

G - specific gravity of particle

K - constant in terminal velocity law; equals 360 numerically

K_1 - constant in F_1 term

K_2 - constant in F_2 term

l - distance fallen by particle

m_p - mass of particle

μ_f - viscosity of fluid medium

ρ_p - mass density of particle

S - distance traveled by particle

t - time

V_o - terminal velocity

V_p - particle velocity

V_{so} - superficial gas velocity in a conduit

F - (subscript) refers to fluid

p - (subscript) refers to particle

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