

# r-Process Nucleosynthesis in Supernovae Simulation

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## ABSTRACT

URRESTA, LYDA A Fortran simulation of the r-process in supernovae, created in order to determine the most likely r-process environment. Department of Physics and Astronomy, June 2011.

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We examined a potential r-process site by modeling nucleosynthesis in a late-stage supernova explosion, considering different hydrodynamic conditions. After collecting the resultant abundances with respect to atomic and mass number, we decayed the isotopes in our data sample to stability. Then, we plotted the decayed and undecayed abundances with respect to their atomic number and mass number, and compared them to the abundances procured from solar and metal-poor halo star data. Both of these environments are known to house r-process elements, and an agreement between the observed abundance data and the abundance data from our simulation would support the theory of a late-stage supernova as a site of r-process nucleosynthesis. Graphing the aforementioned data sets reveals a correspondence between some of the abundance peaks in our simulation and the solar and metal-poor halo star data.

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
1.1	Big Bang . . . . .	1
1.2	Galaxy Formation . . . . .	2
1.3	Star Formation . . . . .	3
1.4	Nuclear Reactions . . . . .	4
1.5	Nucleosynthesis . . . . .	6
1.6	Project Background . . . . .	10
<b>2</b>	<b>Procedure</b>	<b>11</b>
<b>3</b>	<b>Results</b>	<b>12</b>
<b>4</b>	<b>Conclusion</b>	<b>18</b>
<b>5</b>	<b>Appendix</b>	<b>21</b>
5.1	Sample Code . . . . .	21

# 1 Introduction

## 1.1 Big Bang

Prevailing cosmological theory claims that the universe began with the Big Bang, which is believed to have taken place 13.6 billion years ago. At the genesis of this event, the universe was a mass of extremely high density and temperature, estimated to have been at trillions of degrees. Therefore, only subatomic particles (such as protons, neutrons, electrons), neutrinos, and theoretical exotic particles could exist, as the heat would have caused anything larger (i.e., atomic nuclei) to break apart. About 100 seconds after this beginning, the heat lessened to a magnitude of billions of degrees, at which temperature it was possible for protons and neutrons to combine into light nuclei, such as deuterium and some heavier isotopes of hydrogen, helium, and lithium. This early process of assemblage is known as nucleogenesis, or Big Bang nucleosynthesis. A few minutes later, the temperature dropped to about 100 million degrees, at which point nucleogenesis ended, as it was not hot enough for nuclear reactions to continue. At this time, approximately 30% of the universe's mass was in the form of helium. Only the lightest elements were created in the Big Bang, via nucleogenesis, the heavier nuclei forming afterwards.

In the following million years, the process of cooling and expansion continued. Despite this, however, the universe maintained a very high temperature and density, causing it to be opaque. Eventually, the temperature dropped to about 3,000 K, at which point neutral hydrogen atoms began to form. As this process continued, the universe gradually became transparent, though littered with gas clouds in which the first galaxies would begin to form.

## 1.2 Galaxy Formation

At the time of its formation, our galaxy was a large, condensing gas cloud with enough matter to make  $10^{11}$  stars. This matter, composed of mostly hydrogen and helium, was simpler than what we would find presently. The stars that formed from this gas cloud, thus, contained fewer heavy nuclei than stars that formed later on.

As the primeval galaxy came together, it experienced a phenomenon known as spin-up, by which all the matter in a gas rotates around a center as it is pulled inwards by gravity. As spin-up caused the gas cloud to compress, this gas became denser and hotter, the thermal energy coming from the gravitational force. The more rapidly moving gas clouds then began to collide, and their kinetic energy became radiation. As the gas cloud maintained a relatively low density, this radiation was permitted to escape into outer space, which prevented the pressure inside the cloud from rising enough to impede further compression. This process of spin-up continued until the density of the gas cloud was such that the rotational forces, directed outwards from the axis of spin, balanced the inwards gravitational force. At this point, spin up stopped.

The inward pull of gravity then drew the gas cloud together from all directions, causing it to take the shape of a rotating disk on the galactic plane, like the Milky Way we see today. This disk is about 80,000 ly in diameter and 2,000 ly thick. Though most of the galaxy's mass is on the galactic plane, some of it exists in what is known as the galactic halo. The galactic halo is a spherical distribution of stars found in regions above and below the galactic plane and concentrated at the galactic center. As can be expected, these halo stars follow orbits that are not on the galactic plane. Along with the rest of the galaxy, the galactic halo was subject to the spin-up effects present during the galaxy's formation. When considering these effects, we can see that a spin up providing enough rotational force to balance the gravitational pull and prevent a further contraction of the galaxy would have had a rotational force large

enough to tear apart any protostars in its vicinity. The fact that stars were able to form in our galaxy tells us that this rotational energy must have been transferred in some way. Recent findings suggest that rotational energy was transferred via a magnetic field.

### 1.3 Star Formation

Many of the light elements created at the beginning of the universe went on to become part of a gas cloud, like the Orion nebula, kept together by the gravitational attraction between the particles. Clouds like this went on to become the birthplace of stars, which formed inside high density regions. Gravity was partly responsible for the compression of the matter making up these stars. However, gravity alone does not explain their high density. To address this issue, we consider the formation of a protostar.

A protostar, the predecessor of a star, is a large mass formed by the contraction of matter inside a gas cloud. For simplicity's sake, we assume that the protostar is of solar mass, spherically symmetric, and non-rotating. As gravity pulls matter in the protostar towards its center, the radius of the protostar decreases and its density increases. So, at a radius of  $10^{15}$  cm, we would measure a density of approximately  $3 \times 10^{11}$  atoms per  $\text{cm}^3$ . The increased density of the protostar heats its interior, causing atoms inside the protostar to emit radiation. This radiation, however, cannot escape freely due to the high density, but instead bounces from particle to particle in the protostar's interior. Consequently, the interior pressure increases to the extent that it prevents a further decrease in the protostar's radius. At this point, the pressure and gravitational forces are nearly in balance, although not completely so because by the absorption, re-emission, and scattering inside the protostar, some radiation is slowly leaked from the interior to the surface, where it is able to escape freely. Thus, the pressure force is diminished and the process of collapse continues, but at a

significantly slower pace.

The size and density of the protostar adjusts itself until the pressure within each layer is able to balance and support its weight. When each layer of a star achieves this state, we say that it is in hydrostatic equilibrium. As energy is more free to escape from the surface of the protostar than from its interior, matter near the center is hotter than matter near the surface. As the protostar contracts, the temperature in its center increases until nuclear reactions are able to occur. Thinking of atoms as nuclear fuel, equilibrium is reached by the burning of nuclear fuel. When this happens, we say that the protostar has become a star.

## 1.4 Nuclear Reactions

In order to understand the nuclear reactions that take place inside the protostar, we must first introduce the four known forces of nature. From strongest to weakest, they are called the strong nuclear force, the electromagnetic force, the weak nuclear force, and the gravitational force. The strong and weak nuclear forces are short range, while the electromagnetic and gravitational forces spread out towards infinity. For example, in a system of two protons, the repellant electromagnetic force dominates until they get close enough for the attractive strong nuclear force to take over. Note that this strong force is similar for two protons, two neutrons, and a system of proton and neutron, although the only bound combination is proton and neutron. This explains why a stable nucleus having more than one proton requires at least as many neutrons, as these neutrons contribute strong force without the added electrical repulsion. With these concepts in mind, we can begin to talk about nuclear reactions.

Two of the most notable nuclear reactions are nuclear fusion and nuclear fission. In nuclear fusion, two or more nuclei combine to form a single nucleus. In order for the parent nuclei to get close enough to overcome electromagnetic repulsion, they must exist in an environment of high density and temperature. In contrast, nuclear

fission splits a mother nucleus into two or more daughter nuclei.

Another nuclear reaction that allows an unstable nucleus to release energy is nuclear decay, by which the nucleus emits a particle. This may result in a different element or in the same element at a different energy state, depending on the type of particle emitted. The three known types of decay are gamma decay, alpha decay, and beta decay. In gamma decay, energy is released in the form of a gamma particle, which is a high energy photon. A nucleus is thus able to transition from a higher energy state to a lower energy state by emitting a gamma particle, which then carries away the energy difference between the two states. In alpha decay, meanwhile, the particle emitted is an alpha particle, which is a helium nucleus composed of two protons and two neutrons. When the alpha particle gets far enough away from the mother nucleus, the strong force attracting the two falls away and electromagnetic repulsion takes over, accelerating the particle away from the nucleus. Thus, energy in the form of the alpha particle's mass and kinetic energy is carried away from the nucleus. In beta decay, lastly, the nucleus emits a beta particle, which can be either an electron or a positron. This emission creates a change in the charge of the nucleus that depends on the which of the two particles is lost. Additionally, the nucleus emits a neutrino, which is a massless, chargeless particle that travels at the speed of light. Neutrinos can only interact with other particles via the weak nuclear force, which is so weak, and takes so much longer than the strong nuclear force, that a neutrino interaction with matter is rare. The net effect of these emissions is the transformation of a neutron into a proton in the following reaction

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

where  $n$  is a neutron,  $e^{-}$  is an electron, and  $\bar{\nu}_e$  is an antineutrino.

There is an upper bound to the number of neutrons a stable nucleus may have.



Too many neutrons will prompt the beta decay, while too few will be unable to successfully battle the electromagnetic repulsion created by the protons. Therefore, in order for a nucleus to be stable, it must have a specific balance between neutrons and protons. Figure 1 illustrates this required balance in what is known as the valley of stability.

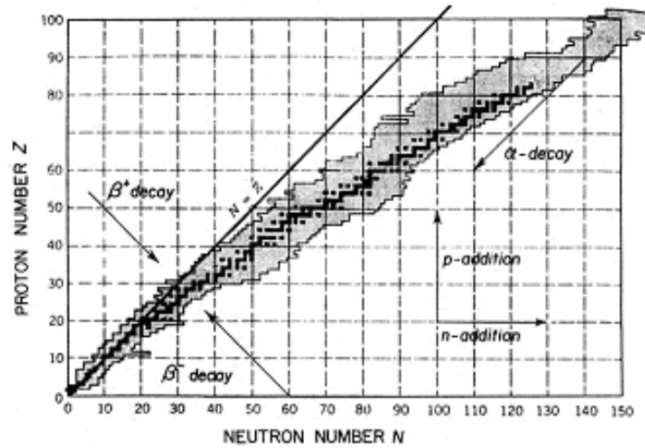


Figure 1: The valley of stability.

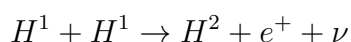
The black dots displayed on the chart create what is known as the line of stability, which shows the stable isotopes of each element, should one exist. The gray portion of the graph, meanwhile, represents all the unstable nuclei that are known to exist. Additionally, the chart shows the type of decay that might occur for unstable nuclei with a given ratio of protons and neutrons.

## 1.5 Nucleosynthesis

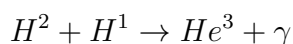
The nuclei shown in Figure 1 are created via nucleosynthesis, which is thought to take place in the interiors of stars and in supernovae. In these environments, nuclei have enough energy to overcome the electromagnetic repulsion, and get close enough for the strong force to dominate. Thus, atomic nuclei are joined to form heavier elements via thermonuclear fusion reactions, releasing large amounts of energy in the process.

In the more massive stars, nuclear fusion releases energy in the production of elements up to iron, and consumes energy in the production of elements heavier than iron.

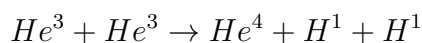
The set chain reactions that produce the heavier elements in stars are called the stellar energy cycles, of which the most common are the proton-proton chain, the CNO cycle, and the triple alpha process. At temperatures less than  $15 \times 10^6$  K in the interior of a star, we typically see the proton-proton chain. In the proton-proton chain, we first have the fusion of two hydrogen nuclei into a deuterium nucleus, a positron, and a neutrino:



The neutrino soon escapes from the star, but the positron collides with an electron to create a gamma ray. Then, the deuterium fuses with hydrogen to form a helium isotope with two protons and one neutron, and a gamma ray:

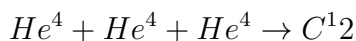


Finally, two of the above helium isotopes fuse to become an alpha particle and two hydrogen nuclei:



In this process, a small amount of mass is converted into energy via the formula  $E = mc^2$ . At stellar interiors hotter than the sun's, the CNO cycle dominates. First, a hydrogen nucleus fuses with a carbon nucleus. After several steps and the insertion of four hydrogen nuclei, the carbon cycle yields a helium nucleus and a carbon nucleus. Since the carbon cycle ends with the same amount of carbon it started with, this cycle can start again. At stellar interiors much hotter than the sun's, where the core's temperature is about  $10^8$  K and all of the hydrogen has fused to helium, three helium nuclei may fuse to make a carbon nucleus in what is known as the triple alpha

process:



Similar chains reactions exist to produce elements up to the weight of iron inside stars.

Elements heavier than iron may be produced by neutron capture processes known as the slow s-process and the rapid r-process. In the s-process, which is thought to occur in red giants, neutron capture occurs more slowly than beta decay, creating nuclei along the valley of stability. So, when repeated neutron capture by a stable nucleus results in an unstable isotope, beta decay occurs before another neutron is captured. The s-process thus produces unstable nuclei no more than one neutron capture away from the valley of stability, creating a path that zigzags near the line of stability. Because the process of neutron capture is so slow, the s-process cannot create very heavy radioactive elements, as they would decay by fission before neutrons could be added. In the r-process, neutron capture occurs faster than beta decay, creating neutron rich material, much of which is unstable. We still do not know where the r-process takes place, although we believe that it would need to be a neutron rich environment of extremely high temperature, such as a supernovae explosion.

One possible r-process environment is conjectured to be the supernovae explosions of massive stars, which are of 10 to 20 solar masses. At the end of the star's life the lack of convection and the outward drop in temperature causes the star to layer into shells of carbon, helium, and hydrogen at an increasing distance from the iron core of the star. Meanwhile, as the temperature increases, the iron core contracts, heating up as it does so. When the core reaches high enough temperatures, the iron begins to photodisintegrate, a process by which an incoming gamma ray knocks a proton or neutron out of the nucleus. This creates an endothermic reaction, a process by which a system absorbs heat energy from its surroundings, which, in this case, uses up about 100 MeV. As the core loses this energy, the pressure created by the increased

heat lessens and the core is able to contract more rapidly. This process of energy loss and contraction accelerates so that within seconds the core collapses and heats up catastrophically. A shock wave forms from the collapse of the outer layers onto the core, and then bounces off to blast through the star. Protons and electrons, meanwhile, fuse to become neutrons, giving off a large number of neutrinos in the process. The outer layers of the star are, at this point, dense enough to stop them. These neutrinos then revive the shock as it stalls at the outer layers, pushing with enough force to blow them off. After this explosion, the star is destroyed and only the core is left behind.

The core of the supernova can then either become a black hole or a neutron star. A neutron star is made almost entirely of neutrons. In general, a free neutron can only exist for about 11 minutes before decaying into a proton and an electron. At a sufficiently high density, however, a neutron does not decay. This results in a large supply of energetic neutrons, which are potentially absorbed by heavy nuclei via the r-process. Some of the material created in these r-process events can be found in metal-poor halo stars. Metal poor stars typically contain only about 3 percent of the heavy elements relative to hydrogen as the sun does. Thus, these stars are underabundant in elements heavier than helium, when compared to the sun. We know this because their spectra have weak heavy element lines. These stars are thought to have been created early in our galactic history, possibly by old supernova explosions, when interstellar gas did not yet contain a substantial amount of heavy elements. As the name suggests, metal-poor halo stars exist in the galactic halo. Abundance patterns with respect to element number in these stars show the existence of r-process nuclei, which were probably left over from some event likely house the r-process, such as a supernova.

## 1.6 Project Background

In order to find a potential environment for the r-process, we study a late-stage supernova explosion. We consider the neutron rich region around the protoneutron star at the supernova's center and study the nucleosynthesis process here. The primary reactions that take place in this environment are

$$n + \nu_e \leftrightarrow p + e^- \text{ and } p + \bar{\nu}_e \leftrightarrow n + e^+$$

where  $n$  is a neutron,  $p$  is a proton,  $e^-$  is an electron,  $e^+$  is a positron,  $\nu_e$  is a neutrino, and  $\bar{\nu}_e$  is an antineutrino. Note that antineutrinos are hotter here than neutrinos, as the large abundance of neutrons relative to protons causes neutrinos to lose energy more often than antineutrinos in the above reactions.

We use a computer model to analyze the process of nucleosynthesis in the neutrino-driven wind of the protoneutron star. First, we consider the hydrodynamics of the expelled material with respect to time. So, the initial code yields the temperature, density, and distance from the center of the protoneutron star of the material with respect to time. The nucleosynthesis model of this material takes place in several steps. First, we know that the protons and neutrons in the wind become alpha particles and then combine to produce heavier elements, up to iron. Then, some time later, when the material is farther from the center and consequently colder, fusion stops and the r-process begins. We switch to a different code to model this r-process and the resultant neutron capture on iron group seeds. This yields a data file containing the abundance of element isotopes with respect to time. After a span of about 10 seconds, we end this process and begin to decay the resultant isotopes to stability.

It is necessary to make this decay to stability in order to meaningfully compare the results of our simulation to the observed abundances in pertinent environments.

In particular, we would like to be able to compare our simulation to the observed abundances in metal-poor halo stars. We have these halo star abundances only in terms of their atomic number, so we need to likewise map our simulation’s abundances with respect to their atomic number. We do this for both the decayed and the undecayed sample, and, as we shall see, the observed abundances of the metal-poor halo star correspond more closely to that of the decayed sample.

## 2 Procedure

Much of our project was centered around Fortran programming. Therefore, it was essential to become well acquainted with Fortran’s basic programming procedures. The book we used for this purpose was Michael Kupferschmid’s *Classical Fortran: Programming for Engineering and Scientific Applications*, which provided a solid introduction to programming with Fortran, acquainting us with several different types of programming techniques in a practical and digestible manner. Kupferschmid’s book, however, addressed arrays at an introductory level and did not delve much deeper than that. As we needed to be able to work with and manipulate arrays of several dimensions in our code, we turned to an additional resource for more information on how to do this. *Fortran 77 for Engineers and Scientists* by Larry Nyhoff and Sanford Leestma supplied the further knowledge we required. For supplementary assistance on basic programming questions, we turned to internet forums on the subject, in which one can pose coding queries at any level. It helped, additionally, to read other people’s questions and the responses to them. Although these questions were generally more advanced than what we were familiar with, they provided a helpful guide on how to address our own issues, or at least pointed us to other resources we could consult.

The Fortran we worked with was installed in Ubuntu, a Unix based operating

system. We used pico as our text editor, which, while it lacks many of the basic functions we are accustomed to seeing on a, for instance, Word processor, is reliable and easy to handle once one becomes used to its nuances. To create our plots, we used gnuplot, which is an open source program designed specifically for this purpose. Additionally, we became well versed in some of the Ubuntu terminal's basic commands, which provide a wide array of time saving shortcuts. When working in any Unix based operating system, it is important to learn its terminal's commands, as this type of parlance with the system makes working with it much easier than it otherwise would be. Once one learns the terminal commands for any Unix based operating system, one can generally apply them to any other Unix based operating system, so it is useful to spend some time doing this.

Once we were relatively comfortable with the technical portion of this project, it was necessary to learn more about the physics it explores. While this may seem obvious, it is nevertheless important to mention, as it is possible to get so engrossed in the programming that one forgets about the physics. If this happens, one may run into an error or an unexpected result, and not understand where it came from or how to fix it. Therefore, knowing about the underlying physics of the programming task one is tackling goes hand in hand with writing better code.

### 3 Results

After decaying the unstable isotopes to stability, we mapped their abundance with respect to atomic mass and atomic number. Then, we compared these graphs with those of the undecayed data sample. For this purpose, we use three different data, where, by varying the hydrodynamics of each process, we simulate the main r-process, the weak r-process, and the strong main r-process. We then compared these graphs with r-process heavy element abundances from solar and metal-poor halo star data.

Note that, in the graphs below, the green line represents the undecayed sample, the red line represents the sample after its decay to stability, the blue crosses represent the solar abundance data, and the pink diamonds represents the metal-poor halo star abundance data.

First, we consider the abundances of both the stable sample and the undecayed sample with respect to atomic mass. Since beta decay does not change an atom's atomic mass, we expect the graphs of the stable and the undecayed samples to look roughly the same. In Figure 2, we see the main r-process abundance with respect to atomic mass, in which this expectation holds, for the most part.

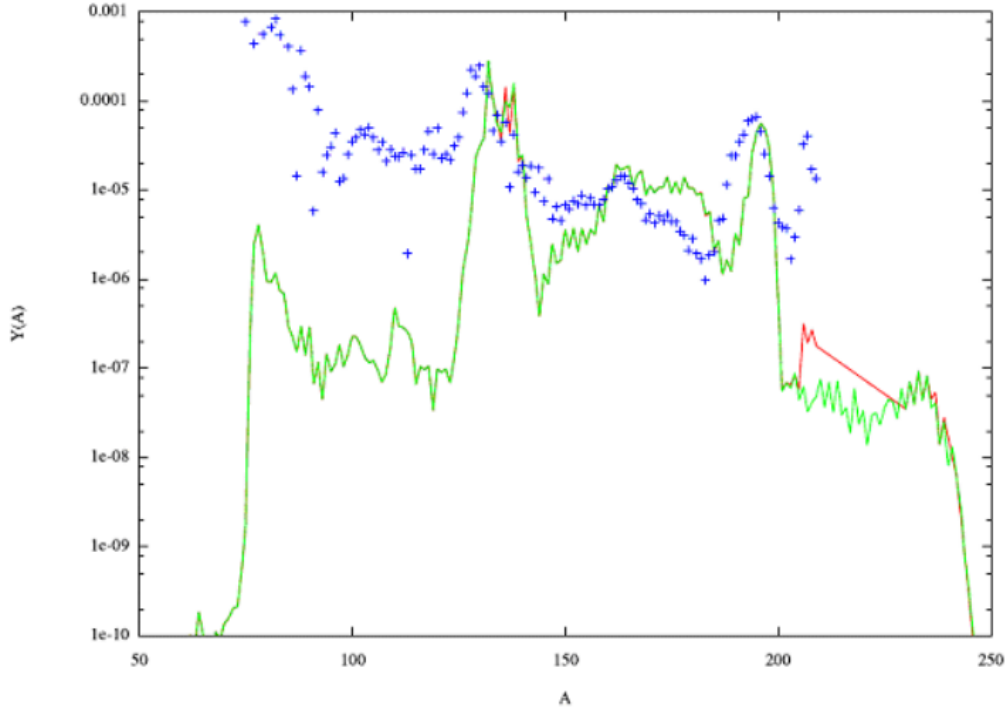


Figure 2: Main r-process abundance with respect to atomic mass.

In the graph above, we see a clear difference between the stable and the undecayed sample abundances with respect to atomic mass for elements heavier than  $A=200$ . This effect is most likely due to alpha decay, which, as we know, changes the atomic



mass of a nucleus. Additionally, between  $A=100$  and  $A=150$ , we note a correspondence between the solar abundance data and the red stable sample abundance peak.

In Figure 3, we have the weak r-process abundance with respect to atomic mass.

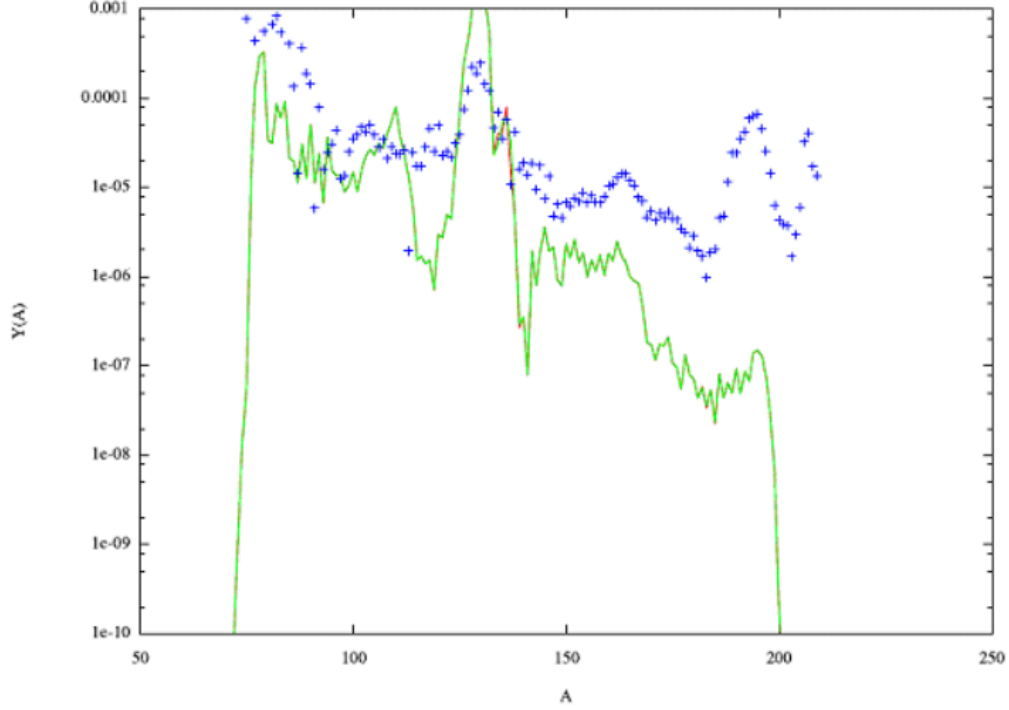


Figure 3: Weak r-process abundance with respect to atomic mass.

Here, we see the first and second abundance peaks, but not the third. As expected, the stable sample and undecayed sample lines concur, except in a few places where beta delayed neutron emission is taking place.

In Figure 4, we have the strong main r-process abundance with respect to atomic mass, where we see a strong third peak.

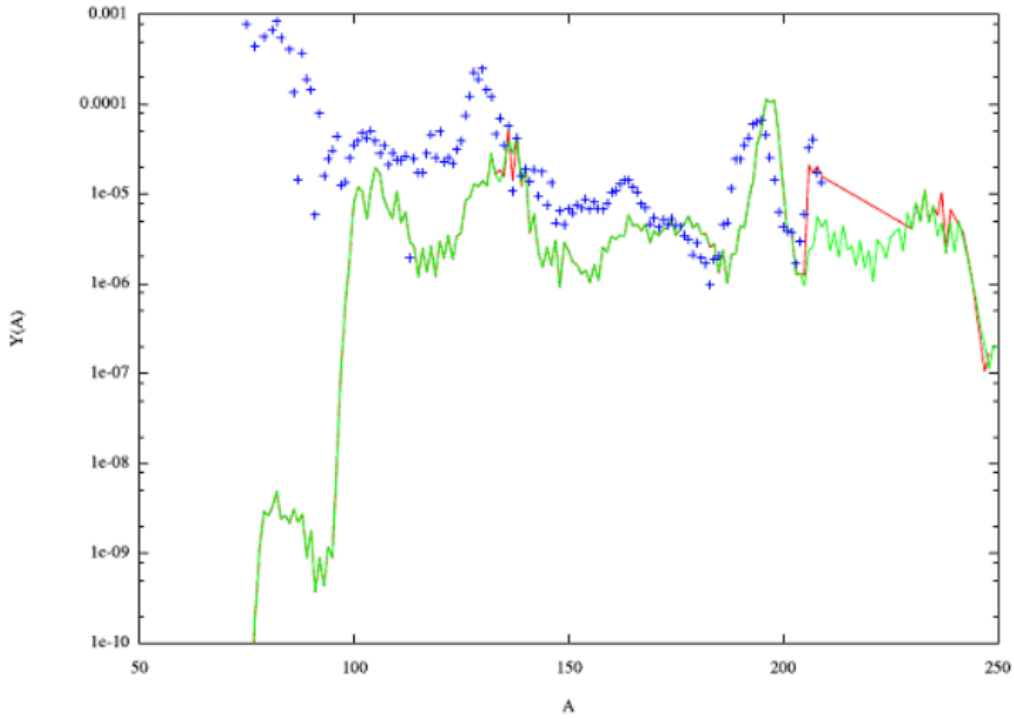


Figure 4: Strong main r-process abundance with respect to atomic mass.

Again, we see that stable sample and undecayed sample lines overlap in most places except for elements heavier than  $A=200$ , where alpha decay, and perhaps some fission, takes place, and for elements between  $A=100$  and  $A=150$ , where beta delayed neutron emission occurs. Notably, we also see a correspondence between the solar abundance data a little after  $A=200$  and the stable sample abundance peak.

Next, we consider the abundances of the stable sample and the undecayed sample with respect to atomic number. Here, we expect the graph to shift after decay to stability, as both alpha and beta decay change the atomic number of an element. In Figure 5, we have the main r-process abundance with respect to atomic number, where this shift is clearly visible.

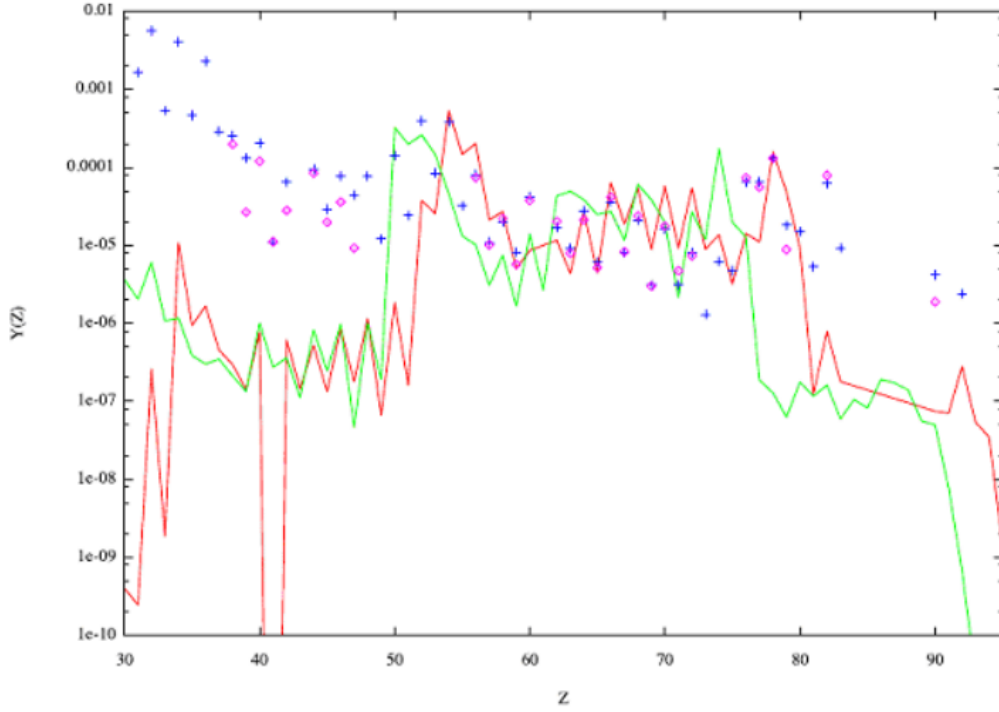


Figure 5: Main r-process abundance with respect to atomic number.

Note that in the graph above that there is an agreement between the stable sample abundance peaks and both the solar and the metal-poor halo star abundances, particularly between  $Z=50$  and  $Z=80$ .

In Figure 6 and Figure 7, we have the weak r-process abundance and the strong main r-process abundance, in turn, with respect to atomic number. Again, we can see the expected shift after decay to stability.

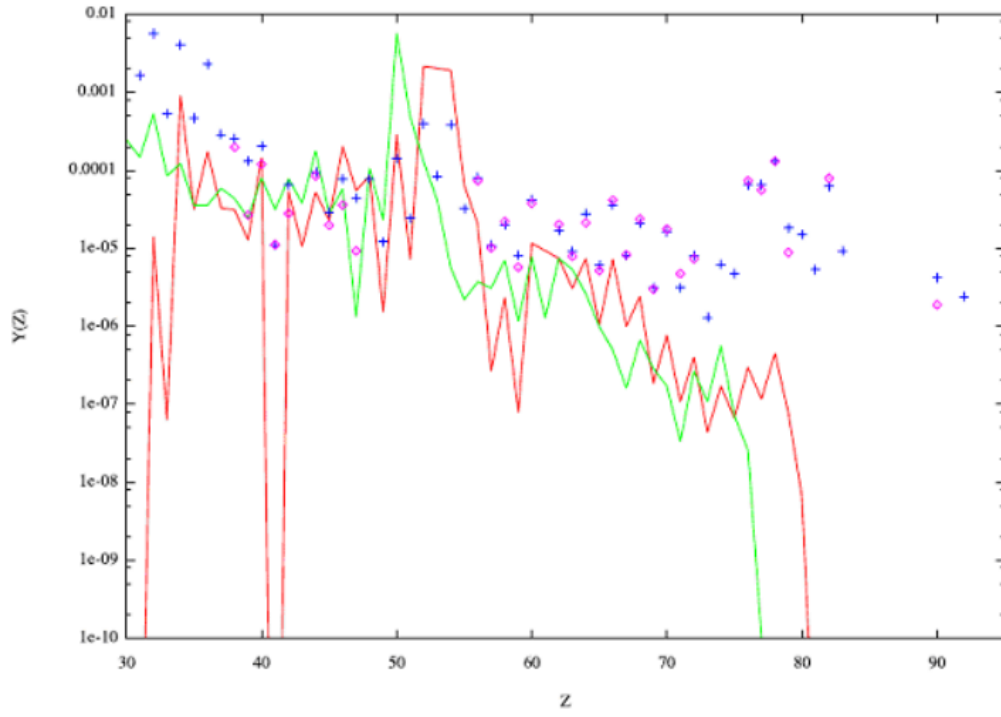


Figure 6: Weak r-process abundance with respect to atomic number.

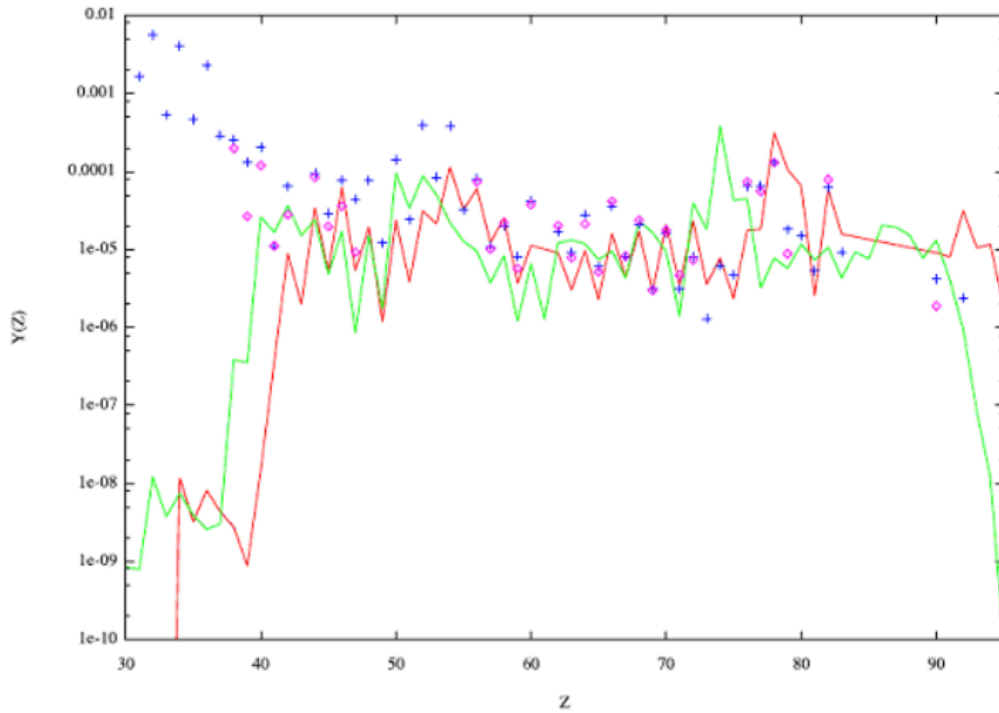


Figure 7: Strong main r-process abundance with respect to atomic number.

We notice that in the graphs displaying the abundance with respect to atomic number (Figure 5, Figure 6, and Figure 7), there is an agreement between the solar and the metal-poor halo star data, which suggests that the r-process nuclei found in these two environments were wrought in a similar, and potentially universal, process. Additionally, we note that many of the peaks and dips in our stable data sample agree with the solar and metal-poor halo star data.

## 4 Conclusion

When comparing the stable sample and undecayed sample data with abundance data procured from solar and metal-poor halo star observations, we see an agreement between the later and the stable sample abundance peaks. Hence, we see that in order to note any significant correlations between the simulation data and the observed

abundances, we need to consider the simulation data after its decay to stability. This is especially true when examining the metal-poor halo star abundance data, of which we only have information in terms of atomic number. So, we would necessarily need to compare it to the simulation's abundance data after its decay to stability. As both the solar and the metal-poor halo star environments are known to house r-process elements, the agreements we see between the observed abundance data and the abundance data from our simulation supports the theory of a late-stage supernova as a site for r-process nucleosynthesis.

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- [11] [http://nedwww.ipac.caltech.edu/level5/Pagel/Figures/figure1\\_2.jpeg](http://nedwww.ipac.caltech.edu/level5/Pagel/Figures/figure1_2.jpeg)

## 5 Appendix

### 5.1 Sample Code

We called the data file containing the nucleosynthesis simulation results YZA\_test.dat, and our coding goal was to decay the element isotopes listed here to stability. For this, we needed a list of the stable isotopes, which we called stable\_nuc.dat. Additionally, we needed information about the type of decay that was likely to occur for each isotope. For this, we used a data file called betpak.dat, containing the probability of beta decay for each isotope, including its probability of emitting zero, one, two, or three neutrons by beta delayed neutron emission. A second file, called alpha.dat, contained the probability of alpha decay for each element isotope. The sum of these probabilities for each isotope was, by requirement, one. With this information, we were able to decay the element isotopes in YZA\_test.dat to stability with the following Fortran code:

```
program bdne_new

  real*8 yold(150,350), ynew(150,350)
  real*8 ap(150, 350), p0(150, 350), p1(150, 350)
  real*8 p2(150, 350), p3(150, 350), mf(50)
  integer stable(150, 350)
  integer i, j, k, iz, ia, r1, r2, r3, r7, r8, r9
  integer sz, sa, a1, a2, az, aa, r10, r11, pz, pa
  real*8 r4, r5, r6, a3, a4, a5, r12, r13, r14
  logical stability
```

c Open the relevant documents.



```

open(unit = 15, file = 'YZA_test.dat')
open(unit = 17, file = 'stable_nuc.dat')
open(unit = 19, file = 'alpha.dat')
open(unit = 21, file = 'betpak.dat')
open(unit = 23, file = 'bdne_new.dat')

c Initialize arrays at 0.
c yold(iz, ia) and ynew(iz, ia): The old and new abundances of each isotope.
c ap(iz, ia): The probability of alpha decay of an isotope.
c p0(iz, ia), p1(iz, ia), p2(iz, ia), p3(iz, ia): The probabilities of beta decay with the
emission of zero, one, two, and three neutrons in the process.
c stable(iz, ia) = 0: The stability status of each isotope. A value of zero means that
the isotope is unstable, while a value one means that it is stable.

do iz = 1, 150
  do ia = 1, 350
    yold(iz, ia) = 0.d0
    ynew(iz, ia) = 0.d0
    ap(iz, ia) = 0.d0
    p0(iz, ia) = 0.d0
    p1(iz, ia) = 0.d0
    p2(iz, ia) = 0.d0
    p3(iz, ia) = 0.d0
    stable(iz, ia) = 0
  end do
end do

c Read YZA_test.dat.

```

```

do 10, i = 1, 1460
    read(15, *) r1, r2, r3, r4, r5, r6
    iz = r2
    ia = r3
    yold(iz, ia) = yold(iz, ia) + r4
10 continue

c Read stable_nuc.dat.
do 20, j = 1, 293
    read(17, *) r7, r8, r9
    sz = r7
    sa = r8
    stable(sz, sa) = stable(sz, sa) + 1
20 continue

c Read alpha.dat.
do 30, k = 1, 494
    read(19, *) a1, a2, a3, a4, a5
    az = a2
    aa = a1
    ap(az, aa) = a5/100.d0
30 continue

c Read betpak.dat.
do 40, k = 1, 4702
    read(21, *) r10, r11, r12, r13, r14
    pz = r10

```

```

    pa = r10 + r11
    p0(pz, pa) = (1.d0-ap(pz, pa))*(r12)/100.d0
    p1(pz, pa) = (1.d0-ap(pz, pa))*(r13)/100.d0
    p2(pz, pa) = (1.d0-ap(pz, pa))*(r14)/100.d0
    p3(pz, pa) = (1.d0-ap(pz, pa))*(100.d0-r12-r13-r14)/100.d0
40 continue

c Make sure p3 is never negative.
    do iz = 1, 150
        do ia = 1, 350
            if(p3(iz, ia) .le. 0.d0) then
                p3(iz, ia) = 0.d0
            end if
        end do
    end do

c Check for the stability of each element.
50 do iz = 1, 150
    do ia = 1, 350

c If the element is stable, then just place its abundance in the array ynew.
        if(stable(iz, ia) .eq. 1) then
            ynew(iz,ia) = ynew(iz,ia) + yold(iz,ia)

c If the element is not stable, then apply the info from betpak.dat to find new abundances (once).
        else

```

```

    if((iz+1 .le. 150) .and. (ia-3 .ge. 1)) then
        ynew(iz+1,ia) = ynew(iz+1,ia) + p0(iz,ia)*yold(iz,ia)
        ynew(iz+1,ia-1) = ynew(iz+1,ia-1) + p1(iz,ia)*yold(iz,ia)
        ynew(iz+1,ia-2) = ynew(iz+1,ia-2) + p2(iz,ia)*yold(iz,ia)
        ynew(iz+1,ia-3) = ynew(iz+1,ia-3) + p3(iz,ia)*yold(iz,ia)
        ynew(iz-2,ia-4) = ynew(iz-2,ia-4) + ap(iz,ia)*yold(iz,ia)
        yold(iz,ia) = 0.d0
    end if
end if

end do

end do

c Rename the ynews as yold again, and check for stability.
stability = .true.
do iz = 1, 150
    do ia = 1, 350
        yold(iz, ia) = ynew(iz, ia)
        ynew(iz, ia) = 0.d0
    end do
end do

c We should make sure there's still some abundance left for a given nucleus.
if(yold(iz, ia) .ne. 0.d0) then
c Check for stability here.
    if(stable(iz, ia) .eq. 0) then
        stability = .false.
    end if
end if
end do

```

```

        end do

c Create a loop of the above process until stability is reached.
    if(stability .eqv. .false.) then
        go to 50
    end if

c Write the resultant stable nuclei and their abundance into bdne_new.dat.
    do iz = 1, 150
        do ia = 1, 350
            if(yold(iz, ia) .ne. 0.d0) then
                write(23, *) iz, ia, yold(iz, ia)
            end if
        end do
    end do

    stop
end

```