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# The History and Mathematics Behind the Construction of the Islamic Astrolabe

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# The History and Mathematics Behind the Construction of the Islamic Astrolabe

Lyda P. Urresta

Submitted in partial fulfilment of the requirements for  
Honors in the Department of Mathematics

UNION COLLEGE

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## ABSTRACT

URRESTA, LYDA The history and mathematics behind the construction of the Islamic astrolabe: An ancient measuring device used to solve problems in the field of astronomy. Department of Mathematics, June 2011.

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In this paper, we examine the mathematical methods employed in the construction of the astrolabe, an ancient measuring device used to solve problems in the field of astronomy. Essentially, the astrolabe is a two dimensional representation of the heavens obtained by projecting the celestial sphere onto the plane. Though several different types of astrolabes exist, our primary focus is on the most popular design, which is created by the stereographic projection of the celestial sphere onto the plane defined by the equator with the south pole as the projection point.

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# 1 Introduction

Mathematics in the Islamic world bloomed between the 8th and 13th centuries C.E., during which time most of Europe was in the midst of what we know today as the ‘Dark Ages’. The Islamic world not only preserved ancient Greek and Indian mathematical knowledge, but expanded upon it. This nurture of learning was largely due to a new emphasis on the research and translation of ancient texts. Thus, many ancient works were preserved and later introduced to the rest of Europe as Arabic translations.

Islamic mathematicians combined the classical Greek and the medieval Indian methods to address some practical issues of the time, many of which had to do with problems in observational astronomy. The Greek approach was primarily geometric, typified by what we are familiar with in Euclidian geometry. The Indian approach, in turn, was largely numerical, introducing notions such as the decimal system and, famously, the concept of zero. By uniting these two techniques, Islamic mathematicians generated the tools necessary to tackle prevalent practical problems. Many of the issues presented to Islamic mathematicians emerged from religious observances, which required them to find a method for calculating significant times and locations. For example, the beginning of a month was defined as the time when the crescent moon is first visible in the western sky. In order to predict when this would happen, Islamic astronomers needed to be able to describe the path of the moon with respect to the horizon. Additionally Muslims needed to be able to determine the direction of Mecca for any given location during each of the five prayer times [3]. These problems required the development of trigonometry, in particular spherical trigonometry, as both the earth and the heavens could be represented as a sphere. The study of spherical trigonometry was not new; the ancient Greek mathematician Menelaos (c. 70-140 CE), famous for his work on geodesics, was the first scholar known to define a spherical triangle ([1], 158-159).

Before venturing into their calculations, Islamic mathematicians had to introduce the conventions and terminology they would use to denote objects of interest. As earth and the heavens were represented by spheres, they first defined important lines and points on the sphere. To start, a *parallel circle* is the circle created by the intersection of the surface of the sphere with a plane that does not pass through its center. A *great circle*, in turn, is created by the intersection of the surface of a sphere with a plane that passes through its center. Islamic mathematicians knew the following facts about great circles: First, given two distinct great circles on the sphere, these bisect each other. Second, for any two diametrically opposite points on the sphere, there exists a unique great circle perpendicular to all the great circles joining these two points. Third, the converse of the above is also true, that is, given a great circle on the sphere, there exist two diametrically opposite points not on this circle such that any great circle through these two points is perpendicular to the given great circle. These two points are then called the *poles* of the great circle. With this terminology in hand, we can state Menelaos’s definition of a spherical triangle, which, in his *Spherica*, he defines to be “The area enclosed by the arcs of (three) great circles on a sphere, each arc being less than a semicircle” ([1], 157-159). This is illustrated in the figure below.

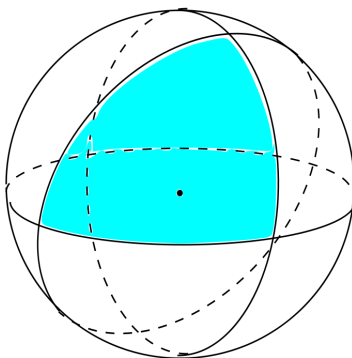


Figure 1: A spherical triangle as defined by Menelaos, where the shaded region represents a triangle created by three distinct great circles on the sphere.

After introducing the above concepts and terminology, Islamic mathematicians applied these to their representation of the heavens and the earth. First, the *celestial sphere* represented the heavens, such that the earth was the center point of the sphere and its surface contained the known celestial objects (such as the sun, the moon, and the stars). We note that although these celestial objects were potentially at different distances from the Earth, they were all considered to be on the surface of celestial sphere, for the purposes of creating a reference. The earth was likewise represented as a sphere, albeit one infinitely smaller than the celestial sphere. The *horizon*, which is the line one perceives to divide the earth and the heavens when looking into the distance, is represented by a great circle on the celestial sphere. The circle is chosen so that when the observer looks directly above him, the point he sees is one of the great circle's poles. This point is known as the *zenith*, and in general represents the observer's location on earth. The great circles perpendicular to the horizon and through the zenith are the *altitude circles*, thus dubbed because given a point above the horizon and an altitude circle through that point, one denotes the altitude of that point as the smaller of the two arcs between the point and the horizon, as shown in Figure 2 ([1], 161).

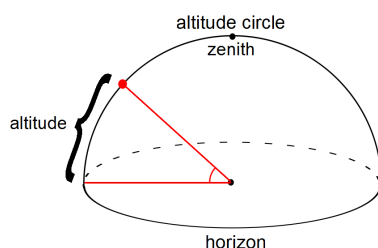


Figure 2: An altitude for a point above the horizon, where the smaller of the two arcs between the point and the horizon represents the altitude of the point.

The altitude circle passing through the north and south points of the horizon is called the *meridian*. The *azimuth* of a point, then, is defined as the smaller angle



between the meridian and the altitude circle of that point, as shown in Figure 3 ([1], 161-162).

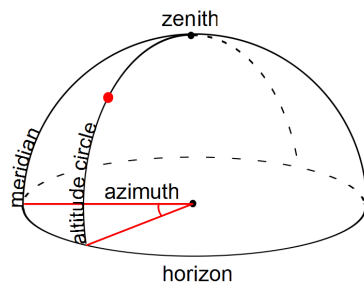


Figure 3: The azimuth of a point, which is defined as the smaller angle between the meridian and the altitude circle of that point.

For any observer, there is a fixed point in the sky around which all celestial objects appear to travel as they rise and set during the day. This point is called the *celestial pole*, and dubbed north pole or south pole according to which hemisphere the observer is located at; for, though two celestial poles exist, an observer can only see one pole from each hemisphere. Note that for all the great circles through the north and south poles there exists a circle perpendicular to all of them, known as the *celestial equator*. Now, as a celestial object rotates around the poles during a 24 hour day, it covers the *day circle*, which is parallel to the equator and intersects the ecliptic (which we will define shortly), and the visible portion of this circle is known as the *arc of visibility*. If an observer stands on the north pole, his perceived horizon is the celestial equator, and his zenith is the north pole. We call any altitude circle defined by such an observer a *declination circle*. The *declination* of an object, therefore, is defined to be the angle created by the smaller arc of the declination circle of the object ([1], 162).

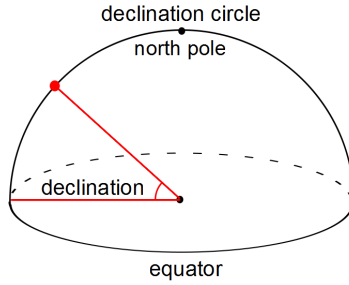


Figure 4: The declination of a point, which is defined to be the angle created by the smaller arc of the declination circle of the point.

Thus, the declination serves as one of the two coordinates by which one can describe the position of a celestial object. The second coordinate is the *right ascension*, alternatively called the *hour angle*, defined as the angle between the prime meridian and the declination circle of the object being observed. That is, it is the azimuth of the object from the perspective of someone on the north pole. The hour angle was thus named because it can be used to tell time: If the observed object is the sun, then we can measure the time until midday by approximating an hour by fifteen degrees and using this to express the hour angle in terms of hours ([1], 162).

If we then proceeded to observe the motion of the sun through the heavens for an entire year, we would see that it travels along a great circle crossing the celestial equator. This great circle is known as the *ecliptic*, and it crosses the equator at an angle of approximately  $23.5^\circ$ . The two points at which the ecliptic bisects the equator are known as the *spring equinox* and the *fall equinox* respectively, so called because day and night are of equal length (hence the use of “equinox”) when the sun is at either of these points. That is, during this time of the year, the day circle is the equator and the arc of visibility is  $180^\circ$ . By convention, we let the  $0^\circ$  point be at the spring equinox and then measure angles counterclockwise from the perspective of an observer at the pole of the ecliptic. Using this method, we can divide the ecliptic into twelve  $30^\circ$  pieces, each representing a *zodiac sign*, a division of the ecliptic traditionally used to

indicate the position of the sun, moon, and planets during a particular time of the year. Figure 5 shows the twelve zodiac signs on the ecliptic ([1], 162-163).

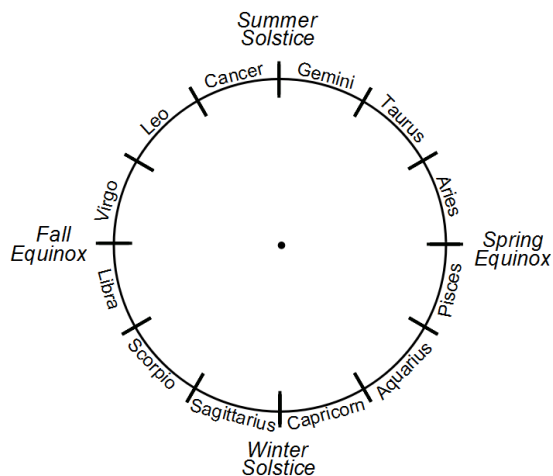


Figure 5: The twelve zodiac divisions on the ecliptic. We show the order of the zodiac signs on the ecliptic, and the position of the equinox and solstice points.

The circle through the northernmost point on the ecliptic, that is, through the point dividing Gemini and Cancer, and parallel to the equator is called the *Tropic of Cancer*. As we see in Figure 5, the point at which the ecliptic and the Tropic of Cancer touch is called the *summer solstice*. Likewise, the circle through the southernmost point on the ecliptic and parallel to the equator is called the *Tropic of Capricorn*, which touches the ecliptic at the *winter solstice*, between Sagittarius and Capricorn ([1], 163).

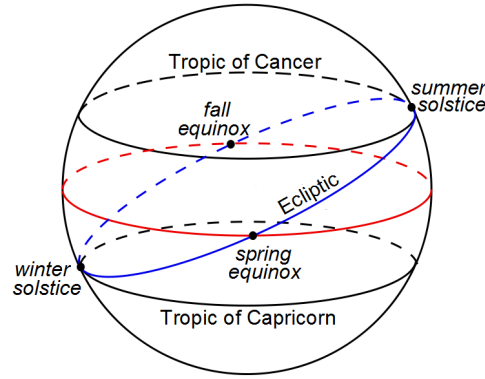


Figure 6: Schematic diagram of the tropics, the ecliptic, the equator, and the equinox and solstice points on the celestial sphere.

In order to analyze astronomical phenomena, it was sometimes necessary to calculate angles or relative distances on the celestial sphere. Doing this directly on the sphere was generally much trickier than it would have been on the plane. Therefore, it was desirable to find an equivalent planar representation for objects on the sphere. Such a representation could be achieved via a *projection*, a mapping of points on the sphere to points on the plane, of which there were several to choose from. We shall see some examples of different types of projections later on. The most commonly used projection, however, was the *stereographic projection*. To create a stereographic projection, we must first choose a plane containing a great circle on the sphere, typically the equator, and a pole of the great circle, typically the south pole. Then, given an object on the sphere that is not on the pole, we draw a straight line between the object and the pole. The point at which this line intersects the our chosen plane is the desired mapping. As the line always intersects the plane at a single point, this map is uniquely defined for every point on the sphere not at the pole. We see a diagram of this projection for some previously mentioned circles in Figure 7 ([11], 6-7).

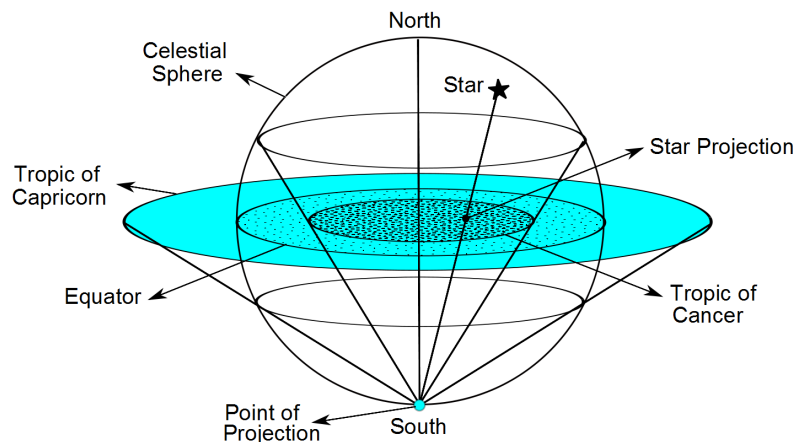


Figure 7: Diagram showing the stereographic projection of points on the celestial sphere from the south pole onto the plane defined by the equator. In particular, we see the projection of the tropics and the equator onto the astrolabe plane. Note that this projection does not produce a uniform spacing for these circles [12].

An important characteristic of the stereographic projection is the fact that it maps circles to circles and preserves angles. This made it possible to tackle problems involving spherical triangles, for instance, using planar trigonometry, which simplified matters considerably. The typical astrolabe, as we shall see, is created by the stereographic projection of celestial objects onto the astrolabe plane.

## 2 The Astrolabe

The *astrolabe* is an ancient measuring device of Greek origin, created at approximately 150 B.C., used to solve problems of spherical astronomy. For this purpose, the astrolabe contains a two-dimensional representation of the heavens created from the stereographic projection of the celestial sphere onto the plane. This projection is done in two parts, that is, as two stereographic projections with each providing particular information about the celestial sphere. Figure 8 shows a typical astrolabe and its different components.



(a) The Astrolabe

(b) Astrolabe Parts

Figure 8: The astrolabe and its parts [13].

In Figure 8, we see that the astrolabe consists of a series of layered brass plates, where the uppermost plate is called the *rete*, the plate underneath is called the *tympan*, and the base plate is called the *mother*. On top of the rete we have the *rule*, a rotating bar used to relate positions on the rete or tympan to the hour scale marked on the graduated edge of the mother, known as the *limb* or *edge*. On the back of the astrolabe, we have the *alidade*, a rule bar with pinhole sights on each end, which is used to measure the altitude of celestial objects in conjunction with the degree scale around the edge ([11], 10-11).

We now examine individual components of the astrolabe.

## 2.1 The Rete

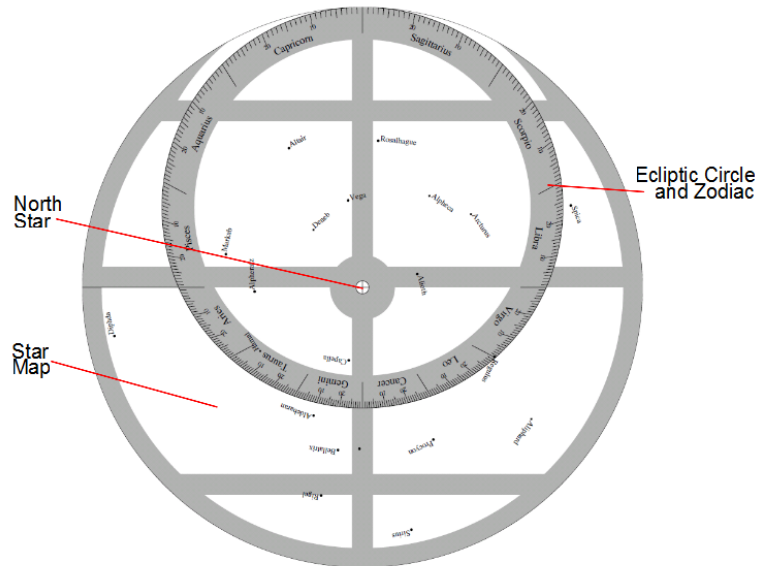


Figure 9: The Rete ([11], 11)

On the rete, we have a collection of pointers showing the locations of brightest stars, forming a star map with the North Star at its center. Additionally, the rete includes the *ecliptic ring*, which is the projection of the ecliptic circle and its twelve zodiac divisions onto the astrolabe plane. The ecliptic ring is used to determine the location of the sun, the moon, and the planets for a given day of the year ([11], 11).

The rete is one of the two projections of the celestial sphere on the astrolabe, and it rotates over the tympan, which is the second projection.

## 2.2 The Tympan

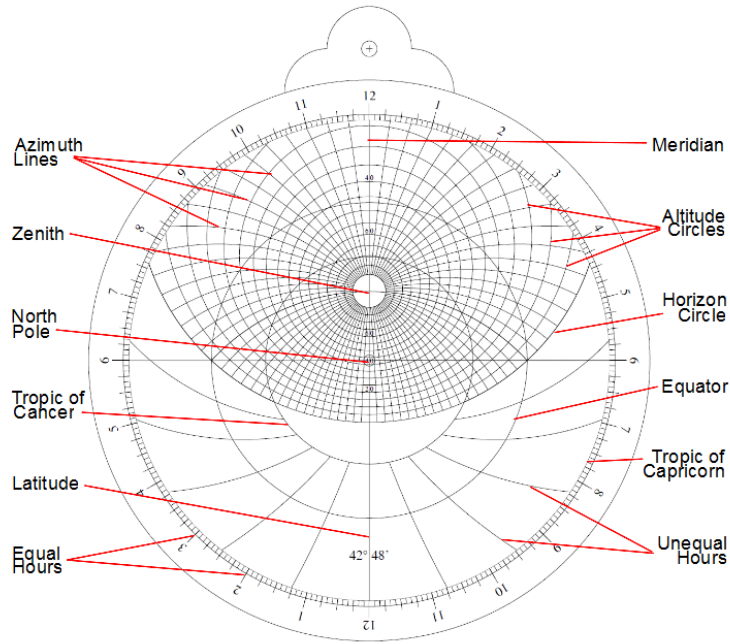


Figure 10: The Tympan ([11], 10)

The tympan shows the principal reference points and lines for objects on the celestial sphere with respect to a specified latitude. These reference points and lines include the zenith, the horizon, the altitude lines (or almucantars), and the azimuth lines. Note that since the tympan is characterized by a particular latitude, a traveler might require several different tympan for the different locations he finds himself at. Additionally, the tympan includes a projection of the equator, the Tropic of Capricorn, and the Tropic of Cancer ([11], 11).

On the limb we see the *equal hours*, with which we can measure time as we are accustomed to doing so today: by dividing the day into 24 equal hours, numbered from 1 to 12 twice. The region on the tympan under the horizon, in turn, shows the *unequal hours*, which divides daytime into 12 equal parts and nighttime into 12 equal parts. Note that the unequal hours are equivalent to the equal hours during



the equinoxes, when day and night are of equal length and hence their divisions are also equal ([7], 219).

## 2.3 The Back of the Astrolabe

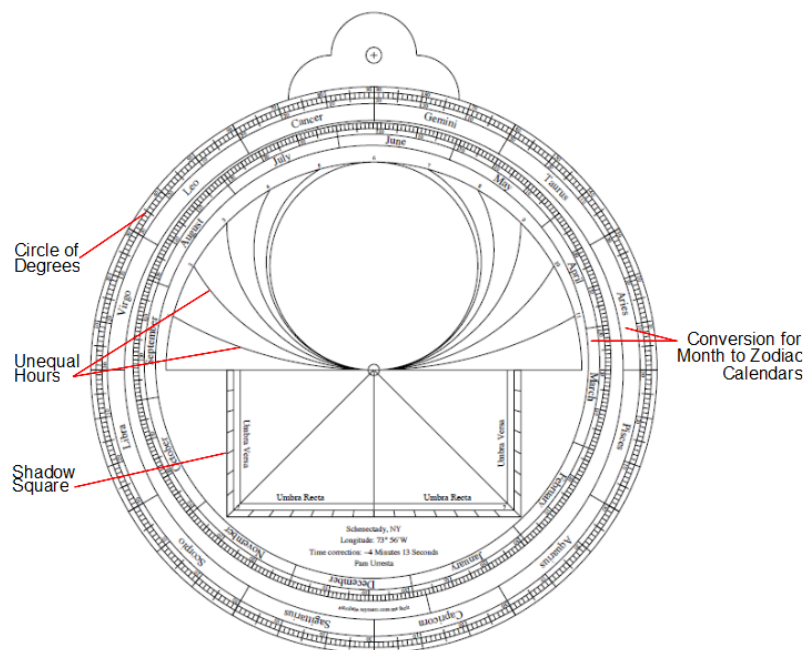


Figure 11: The Back of the Astrolabe.

The back of the astrolabe is typically used to make altitude measurements of celestial objects by using the circle of degrees around the edge. We can also determine the position of the sun for any given day of the year by using the appropriate zodiac conversion displayed near the circle of degrees.

Additionally, the back of the astrolabe includes the *shadow square*, which is the scale typically used by a surveyor to solve problems involving tangents and cotangents. As we can see in Figure 11, the shadow square is shaped like a rectangle and divided into two equal squares, where the left, right, and lower sides of each square have uniform division marks. The point at which the alidade crosses the shadow square after a measurement gives angle information in terms of fixed ratios on the sides of

the triangles created, that is, in terms of tangents and cotangents ([4], 184). Later on, we shall see an example of how to use the shadow square.

The upper three components of the astrolabe, that is, the rule, the rete, and the tympan, sit upon the mother. The back of the astrolabe, in turn, can be found behind the mother. The astrolabe typically includes a ring and shackle used to hang the device vertically.

### **3 Use of the Astrolabe**

If one is imaginative enough, one can think of a myriad of uses for the astrolabe. Here are a couple of the most common.

#### **3.1 Finding the Height of an Object**

To determine the height of an object, we must raise the astrolabe to eyelevel, holding it by the ring, and point the alidade on the back of the astrolabe in the direction of the object. We can then use a shadow square with  $n$  divisions to create the similar triangles shown below.

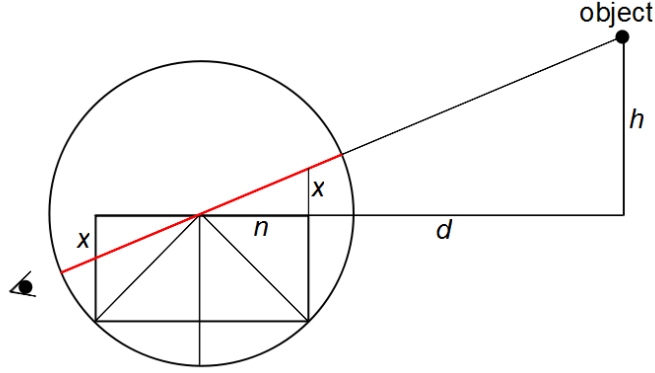


Figure 12: Finding the height of an object using the shadow square, letting the black dot be the object, the red line be the alidade,  $h$  be the height of the object,  $d$  be the distance between the object and the observer,  $n$  be the number of divisions on a side of the shadow square, and  $x$  be the division number one reads on the shadow square after pointing the alidade towards the object.

We can apply similarity of triangles to write  $\frac{x}{n} = \frac{h}{d}$ , which gives us the height  $h$  of the object

$$h = \frac{xd}{n},$$

as desired ([10], 225-226).

### 3.2 Finding the Time of a Celestial Event

Without loss of generality, we focus here on finding the time of sunrise. We can then apply the same principle to find the time of any other celestial event.

For this measurement, we will need to use the front of the astrolabe. First, we must find the location of the sun on the ecliptic for the day of the measurement. Then, we must point the rule towards the sun's position on the ecliptic, and rotate both the rule and the rete until their intersection crosses the eastern horizon (the location of sunrise). Reading the time the rule now points to gives us the desired measurement.

## 4 The Geometry of the Astrolabe

### 4.1 Projection Radius of a Point on the Celestial Sphere Whose Declination is Known

#### 4.1.1 Projection Radius of the Tropic of Cancer

We can draw the lines, points, and circles on the astrolabe using geometric and trigonometric techniques. First, we show a method for finding the projection radii of the tropics on the astrolabe in terms of the radius  $r$  of the circle representing the equator on the astrolabe. To start, we find the projection radius  $x$  of the Tropic of Cancer using the diagram shown below.

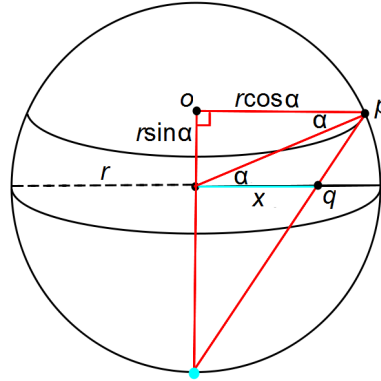


Figure 13: Diagram of the method for finding the radius  $x$  of the circle of the Tropic of Cancer on the astrolabe, given radius  $r$  of the equator.

We choose point  $p$  on the Tropic of Cancer and draw a line from this point to the south pole, so the stereographic projection of the radius of Tropic of Cancer is given by the intersection of this line with the equatorial plane at point  $q$ . The radius of projection is then denoted by  $x$ . Then, we draw a line from the origin (the center of the Earth) to  $p$ . This line is a radius of length  $r$ . We then draw the radius of the Tropic of Cancer and intersect it with a straight line going through the south pole and

the origin. This creates the triangles shown in Figure 13. We can then solve for  $x$  by using similar triangles to show

$$\frac{r}{x} = \frac{r + r \sin \alpha}{r \cos \alpha}$$

which gives us

$$x = \frac{r \cos \alpha}{1 + \sin \alpha},$$

the projection radius of the Tropic of Cancer in terms of  $r$ .

#### 4.1.2 Projection Radius of the Tropic of Capricorn

Using a similar method, we can find the projection radius  $y$  of the Tropic of Capricorn, as shown by the diagram below.

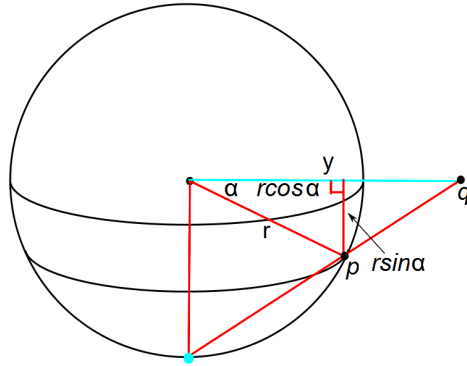


Figure 14: Diagram of the method for finding the radius  $y$  of the circle of the Tropic of Capricorn on the astrolabe, given radius  $r$  of the equator.

As before, we choose a point  $p$  on the Tropic of Capricorn and draw a line from this point to the south pole, so the stereographic projection of the radius of the Tropic of Capricorn is given by the intersection of this line with the equatorial plane at point  $q$ . The radius of projection is then denoted by  $y$ . Then, we draw a line from the

origin to point  $p$ , which, again, is a radius of length  $r$ . We then add a line from the south pole to the origin and another line parallel to it through point  $p$ . We can use similar triangles again to show

$$\frac{y}{r} = \frac{y - r \cos \alpha}{r \sin \alpha}$$

which gives us

$$y = \frac{r \cos \alpha}{1 - \sin \alpha},$$

the projection radius of the Tropic of Capricorn in terms of  $r$ .

Note that we could have alternatively found  $y$  by letting  $\alpha = -\beta$

## 4.2 Drawing the Rete on the Astrolabe

### 4.2.1 Projecting the Ecliptic

We can determine the projection of the ecliptic on the astrolabe by noting that the solstice points, which define a diameter of the ecliptic, can be found on the Tropic of Cancer and the Tropic of Capricorn respectively. Therefore, the projection of the ecliptic onto the astrolabe should have a diameter whose endpoints lie on each of the two circles defining the tropics, as we see in Figure 15.

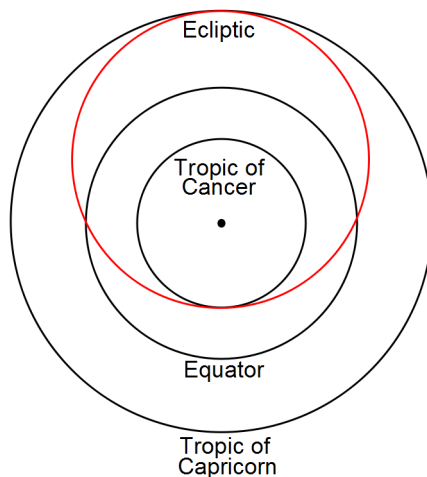


Figure 15: The equator, the tropics, and the ecliptic as they appear on the astrolabe.

#### 4.2.2 Mapping the Stars

In order to find the positions of the brightest stars on the astrolabe, we need information about their declinations and right ascensions. Ancient Islamic astronomers had tables detailing these values for the main stars. We can use the declination  $\delta$  of the star to find the distance  $d$  of the star from the center of the astrolabe with the equation

$$d = \frac{r \cos \delta}{1 + \sin \delta},$$

where we again let  $r$  be the radius of the equatorial circle on the astrolabe. We see this in Figure 16.

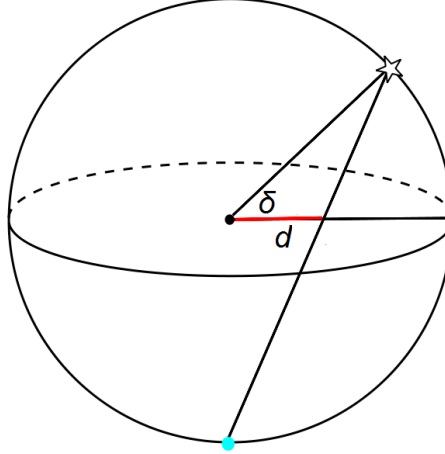


Figure 16: Distance  $d$  of a star with declination  $\delta$  from the center of the astrolabe.

Note that the equation for  $d$  is just a generalization of the radii projections we did above. The right ascension, in turn, is invariant in this mapping, as it is just the angular distance along the equator whose center is the center of the plane of projection. Thus, the angular component of the projection coordinates is just the right ascension. We can then use a protractor and the right ascension of the star to find its angle along the determined radius  $d$ , centering the protractor at the center of the astrolabe and letting the  $0^\circ$  point be at the spring equinox. Using this method, we are able to map the desired stars onto the astrolabe. These stars typically include: Aldebaran, Altair, Arcturus, Bellatrix, Betelgeuse, Deneb, Sirius, Spica, Vega, and others, depending on the particular astrolabe ([8], 5).

#### 4.2.3 Dividing the Ecliptic

We can divide the ecliptic circle into its twelve zodiac signs using the geometric technique shown in the figure below.



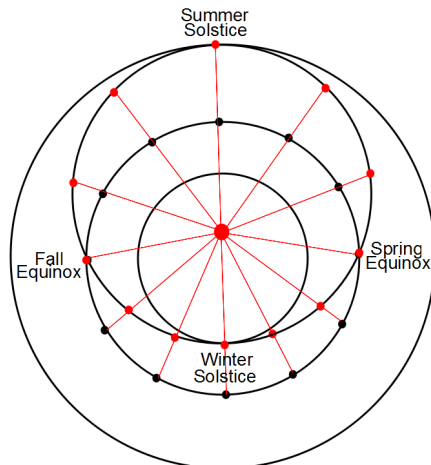


Figure 17: Diagram of the method for dividing the ecliptic on the astrolabe into its twelve zodiac signs, where the ecliptic pole is represented by the large red dot, and the divisions of the ecliptic are represented by smaller red dots.

Note that since the plane defined by the ecliptic on the celestial sphere is not parallel to the plane defined by the equator, the divisions of the ecliptic on the rete are not uniformly spaced. To determine these divisions, we first divide the equatorial circle into twelve equal parts of  $30^\circ$ , starting at the spring equinox point. Then, we draw lines through the ecliptic pole and each of these divisions. The divisions on the ecliptic are marked by where these lines intersect the ecliptic ([11], 8).

We now move on to some ancient techniques for mapping particular lines, points, and circles on the astrolabe.

## 4.3 Ancient Methods

### 4.3.1 Ptolemy's Method

Ptolemy was the first writer to provide a method for dividing the ecliptic, which he did *Planisphaerium*, his text on the astrolabe. First, he notes that the straight line joining the solstices and straight line joining the equinoxes on the astrolabe intersect to divide the astrolabe into four equal parts, as shown below.

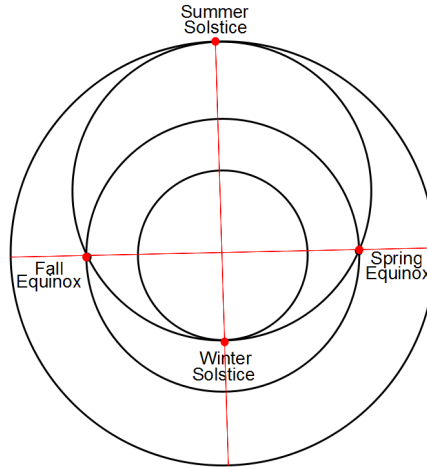


Figure 18: The astrolabe can be split into four equal parts by drawing a line through the solstice points, and a line through the equinox points.

Therefore, in order to divide the ecliptic, we can just construct four circles parallel to the equator such that each of these circles crosses two division points on the ecliptic, as shown in Figure 19.

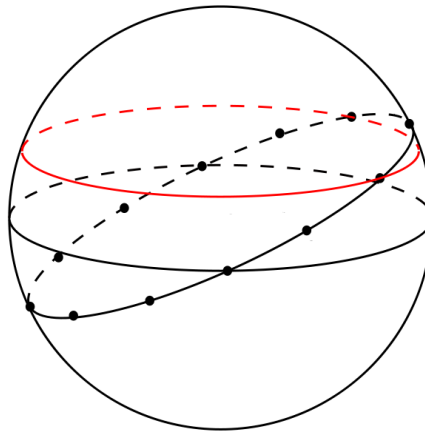


Figure 19: The circle shown in red is parallel to the equator and goes through two division points on the ecliptic.

Thus, we can find the divisions on the ecliptic by projecting these parallel circles onto the astrolabe and finding where they intersect the ecliptic circle. These are our

desired points of division ([2], 310-311).

### 4.3.2 Mapping the Unequal Hours

We now describe a method for mapping the unequal hour divisions on the astrolabe. First, we consider an arbitrary horizon. On any given day of the year, we can draw a day circle through the sun's position, which we recall must be parallel to the equator. We see that  $1/12$  of the day circle's arc of visibility is a day hour, and  $1/12$  of the remainder of the circle is a night hour. Without loss of generality, we find the unequal hour line for the first hour of night. To do this, we consider the path of the sun at night, that is, the part of the day circle below the horizon. Then, we divide the nighttime portion of the day circle into twelve equal parts and consider the first division, that is, the first hour of night, as shown in Figure 20.

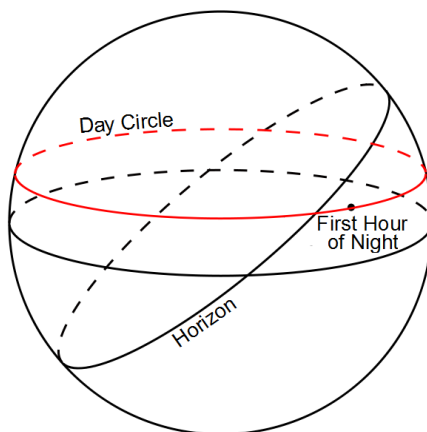


Figure 20: We find the first hour of night for the given horizon, using the day circle for an arbitrary day of the year.

Now, if we find the first hour of night when the sun is on the Tropic of Cancer, on the equator, and on the Tropic of Capricorn, we can project these three points onto the astrolabe plane and draw a unique circle through them. The arc of this circle contained in the astrolabe is the unequal hour arc of the first hour of night. We can

find the other unequal hour lines by using an exactly similar method ([1], 171-176).

#### 4.3.3 Great Circles Through the Poles Become Straight Lines on the Astrolabe

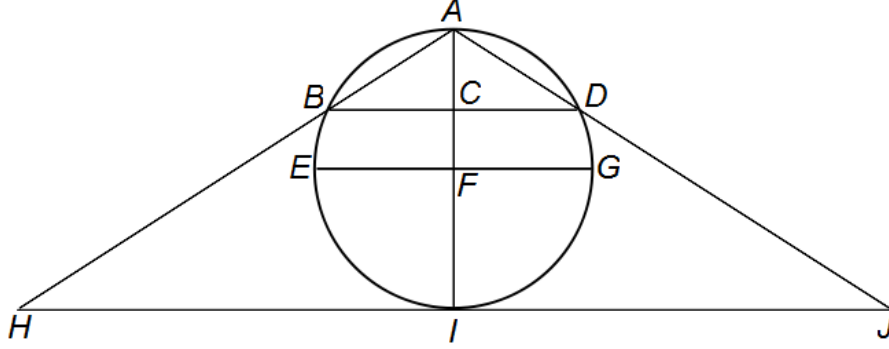


Figure 21: Diagram of meridian  $AEIG$ , with center  $F$ , south pole  $A$ , north pole  $I$ , equator  $EG$ , Tropic of Capricorn  $BD$ , great circle intersecting at the poles  $AI$ , and projection plane  $HJ$ . In this projection, we map points on the celestial sphere to a plane tangent to the celestial sphere instead of through it

Using the diagram above, Al-Farghani, a 9th century astronomer whose research included the study of the astrolabe ([9], 222), shows that any great circle going through the poles of the equator (the meridians) is mapped to a straight line on the astrolabe. We let the meridian be described by the circle  $AEIG$  with center  $F$ . Additionally, we let  $A$  be the south pole,  $I$  be the north pole, line  $EG$  be the diameter of the equator circle, and line  $BD$  be the diameter Tropic of Capricorn circle. Now, we let  $HJ$  be a plane tangent to circle  $AEIG$  at point  $I$ . By stereographic projection from projection point  $A$  to projection plane  $HJ$ , we see that we can represent the plane of the astrolabe by a circle on the plane with center  $I$  and diameter  $HJ$ . Note that in this projection, we map points on the celestial sphere to a plane tangent to the celestial sphere instead of through it. This is still a valid stereographic projection, albeit an unusual one.

Now, we let  $A$  be fixed. If we rotate line  $AI$  around circle  $AEIG$ , we see that it

passes in the plane of the astrolabe through line  $HJ$ , as shown in Figure 4.3.3.

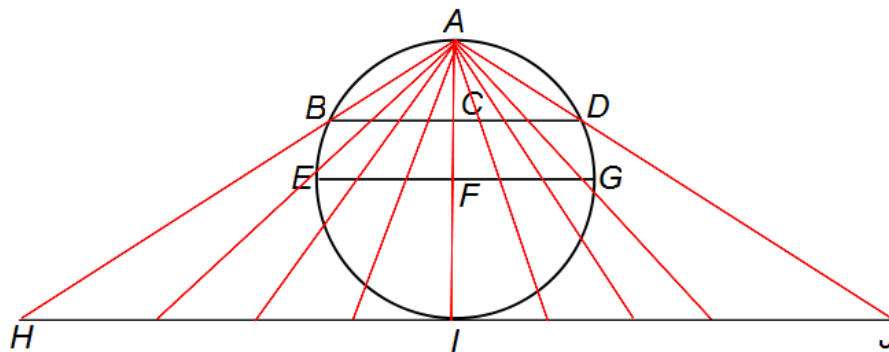


Figure 22: We let  $A$  be fixed and rotate line  $AI$  around circle  $AEIG$ .

That is, the stereographic projection of circle  $AEIG$  onto the astrolabe plane is a straight line. Since all great circles that intersect at the poles of the sphere can be dealt with in an identical manner, we see that all such great circles will form a straight line on the astrolabe ([6], 41-47).

Note that this proof still holds if we use the projection plane defined by the equator. In fact, it holds for any plane we decide to use.

## 5 Alternative Astrolabe Designs

Though the most common astrolabe is described above, other astrolabe designs existed as well.

## 5.1 The Universal Astrolabe



Figure 23: The Universal Astrolabe

The Universal Astrolabe consists of a single plate that can be used for any location (hence the name “universal”) with a graduated pointer with cursor on the front. In this astrolabe, the stereographic projection is not done from an equitorial pole, but from the spring equinox point on a plane through both equitorial poles ([8], 10).

## 5.2 The Spherical Astrolabe



Figure 24: The Spherical Astrolabe

The spherical astrolabe has a rete shaped like a spherical shell, where the stars and ecliptic are located. The ball enclosed under the rete holds several important location markers, such as the horizon and the altitude lines. A user can adjust this astrolabe

to the desired latitude using holes in the ball to access the pivot that rotates the instrument ([11], 3).

### 5.3 The De Rojas Astrolabe



Figure 25: The De Rojas Astrolabe

The De Rojas astrolabe stereographically projects points on the celestial sphere using a projection point at infinity on the line through the equinoxes. This is an example of an orthographic projection in which the parallels on the celestial sphere are projected onto straight lines parallel to the equator ([11], 4).

### 5.4 The Mariner's Astrolabe



Figure 26: The Mariner's Astrolabe

The Mariner’s astrolabe cannot really be called an astrolabe in the conventional sense. It can be categorized more accurately as a sighting instrument, with which mariners could measure the position of stars above the horizon. This instrument was designed to hang vertically, and made heavy to minimize the effect of movement or wind upon it. At the center of the heavy bronze disk framing the astrolabe is a pivot about which the sighting bar can rotate. Note that the typical astrolabe generally incorporated a sighting bar as well ([11], 5).

We now take a closer look at another alternative astrolabe design, the Melon-Shaped astrolabe.

## 6 The Melon-Shaped Astrolabe

The oldest known reference to the melon-shaped astrolabe is a criticism against it made by Al-Farghani. He complained about the cumbersome projection that defines the melon-shaped astrolabe, claiming that it is inordinately difficult to draw. It is perhaps because of this complicated mapping that no example of a melon-shaped astrolabe has yet been found ([5], 2).

The projection used for the melon-shaped astrolabe is called the “azimuthal equidistant projection”, also known as the “zenithal projection”, which maps the entire celestial sphere minus the south pole within a circle of finite radius. Figure 27 shows an example of this projection for the celestial sphere with the south pole as the projection point.



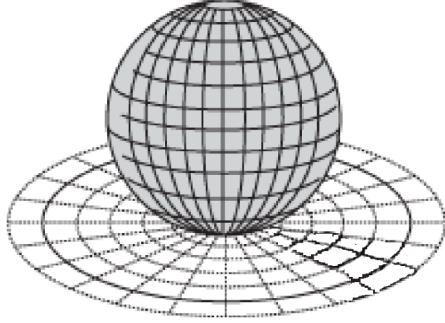


Figure 27: An azimuthal equidistant projection of the celestial sphere from the south pole.

As we can see, this projection maps meridians to straight lines through the projection pole at the center. Circles, in turn, do not map to circles, but to transcendental curves. The defining property of this projection is that it preserves the distance between the pole and any other point on the sphere, such that drawing a meridian line through the point and measuring the resultant radius on the projection yields the correct distance between the pole and the point on the sphere [14].

## 7 Conclusion

The Islamic astrolabe is an intricate, mathematically rich device that was used for centuries as a primary tool for research in astronomy. It was a measuring tool for celestial objects and events, and its uses included the measure of celestial altitudes, the measure of time, the measure of location, etc. At its core, the astrolabe is a two dimensional representation of the heavens obtained by projecting the celestial sphere onto the plane. While there are several types of projections seen on Islamic astrolabes, the most popular is the stereographic projection.

Much of the ancient work on the astrolabe was dedicated to developing techniques for mapping particular lines and points on the celestial sphere onto the astrolabe plane, a study that has spanned over several centuries. Today, the astrolabe stands

as part of the legacy of Islamic mathematics, and is both the product and the origin of several lifetimes' worth of mathematical research.

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