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Applying Fair Division to Global Carbon

Emily D. Lacroix
Union College - Schenectady, NY

Applying Fair Division to Global Carbon Emission Permit Markets

by Emily LaCroix

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ABSTRACT

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The world climate policy debate has come to a political standstill between developed and developing countries. They cannot agree on a "fair" manner to decide how much each country is allowed to pollute, and who should pay for pollution abatement costs. The United States and developed countries believe that all countries should participate and reduce their carbon dioxide emissions to their 1990 levels because everyone will benefit. By contrast, developing countries believe that developed countries should be required to do the majority of the emission abatement because they cause the majority of the pollution.

Carsten Helm [2008] proposed an unconventional emission trading scheme that uses fair division (or cake-cutting), a mathematical tool for dividing up a common resource in a manner that the recipients believe is fair. Helm sets out four equity criteria to be met by a fair division of pollution allowances–Pareto efficiency, individual rationality, stand-alone upper bound, and envy-freeness. He developed a fair division method, the "bounded Walrasian solution," to meet these criteria. Each countries appropriate transfer payment is determined from its marginal abatement cost curve and its initial pollution allowances. With Helm's method, all countries participate in pollution abatement, but it turns out that developing nations are fully compensated for their incremental abatement costs through emission trading. The key difference between Helm's scheme and a conventional cap-and-trade system is the stand-alone upper bound, which imposes that no country should be better off than if it consumes the entire common resource.

My thesis examines Helm's application of fair division equity criteria to the division of emission permits. I use actual carbon dioxide emission data and estimated marginal cost abatement data to simulate a market with a subset of ten countries. I apply Helm's emission trading scheme to the simulated market and observe the results. I analyze the market outcome and discuss whether this is a feasible solution for controlling global carbon dioxide pollution.

Contents

List of Tables

Chapter 1

Indroduction

The world climate change debate is everything but a simple matter. It crosses almost every discipline, and has the possibility of affecting the entire world. With so many different issues and opinions it is easy to understand why the climate policies have come to a political standstill in recent years. Global warming is human-induced climate change through the accumulation of CO_2 and other greenhouse gases in the atmosphere since industrialization. Even though climate change is a natural process, the changes that are predicted to occur within the next 100 years will be extremely rapid; the IPCC believe that the average global temperature increase will be from 1.8 to 4.0° [IPCC 2007]. The effects of global warming are already being felt with the retreat of glaciers, weather related disasters, and changing precipitation patterns, but they will get significantly worse if nothing is done to mitigate greenhouse gas emissions.

Global warming is the result of a market failure. Greenhouse gas emissions are externalities because the emitter does not internalize the full costs, like climate change. Since the climate is a public good (non-exclusive and non-rivalrous), markets do not automatically create a socially optimal equilibrium. Without regulation, individuals and firms will exploit public goods. This means that independent individuals and firms behave in their own self-interest consuming and inevitably depleting the public good, even though it is in the collective long-run interest of the group to not deplete the resource below its sustainable level. In global warming, the climate and environment are suffering from the tragedy of the commons. It is in everyone's interest to stop climate change in the long run, but self-interest leads countries to still increase greenhouse gas emissions because there is no incentive for polluters to reduce their emissions for fear of free riders. Free riders are those who benefit from the actions of others, without contributing. In terms of global warming, countries are unwilling to reduce emissions to slow global climate change, if other (free-riding) countries are not willing to contribute, as well. The free rider countries would enjoy the benefits of slowed climate change, but not have to do the work.

In order to encourage pollution abatement and internalize emission costs, governments and intergovernmental organizations (IGOs) implement market-based approaches, like cap-and-trade and carbon taxes, to provide economic incentives. Governments and IGOs must fill the important role of regulator to counteract the global warming market failure. These regulatory bodies set an abatement target–a certain amount of global emission reduction–that could be met in a number of ways, including through the market-based approaches mentioned above. This abatement target should eventually bring greenhouse gas emissions back down to a reasonable level, where climate change is slowed or stopped.

Ever since the Kyoto Protocol the world climate policy debate has come to a political standstill with countries unwilling to fully commit to their individual abatement targets. They cannot agree on a "fair" manner to decide how much each country is allowed to pollute, and who should pay for pollution abatement costs. My thesis examines an unconventional emission trading scheme proposed by Helm [2008]. It applies fair division (or cake-cutting) properties to the division of emission allowances in order to come up with a "fair" final allocation of the common resource that all participants agree upon. These properties (or equity criteria) are Pareto efficiency, individual rationality, envy-freeness, and the stand-alone upper bound. The key difference between Helm's scheme and a conventional cap-and-trade system is that it satisfies the stand-alone upper bound, which requires that no country should be better off than it would be if it consumes the entire common resource.

This thesis examines Helm's application of the fair division equity criteria. We use actual carbon dioxide emission data and estimated marginal cost abatement data to simulate a market. We then apply Helm's emission trading scheme to a simulated market of 10 countries and observe the results. We found that Helm's scheme creates a significantly different market outcome than a conventional cap-and-trade scheme. Surprisingly, we found that both per capita and proportional initial entitlements lead to violations of the stand-alone upper bound, and require a redistribution of utility among countries. We also suggest that there should be further research into other justifiable rules for the division of initial entitlements between agents. Although we assume that per capita and proportional initial entitlements are justifiable rules, most likely there is still a more appropriate method for dividing initial entitlements, like proportional to current emissions.

Chapter 2 of this paper discusses the background of global warming, its debate, and current and potential abatement programs. Chapter 3 discuss the literature on abatement programs, focusing on their respective efficiencies and issues. Chapter 4 is an in-depth description and discussion of Helm's abatement program and the model to be used. Chapter 5 describes the data and measure to be used in the global emission allowance market simulation. Chapter 6 discusses the results from the simulation. The paper concludes with Chapter 7, which discusses the feasibility of Helm's bounded Walrasian solution as an answer to fair global abatement efforts. Lastly, the appendix holds all the data and results tables.

Chapter 2

Background

This chapter discusses background information about climate change and possible mitigation policies. This background is necessary to fully understand the problems that climate change pose, and the issues with current carbon reduction methods that are proposed as potential solutions.

2.1 The Debate

Global warming is going to be extremely expensive to prevent or even slow. The question is who will pay for and participate in the pollution abatement process. The fact of the matter is that some developed countries, like the United States, emitted the majority of greenhouse gases, but now they argue that all countries should participate in the pollution reduction because everyone will enjoy the benefits. They are concerned about free riders, who will receive the benefits of greenhouse gas reductions while not bearing the costs. The United States (during the Clinton and G.W. Bush years) was especially wary of excluding growing developing countries, like China and India, from international pollution regulation agreements because they will be the next big emitters. This is why the Senate did not ratify the Kyoto Protocol in 1997, and President George W. Bush withdrew the United States' support for the Protocol in 2001.

On the other hand, developing countries want the freedom to grow by the fastest means possible and not be constricted by restricting or reducing their greenhouse gas emissions. Many were hesitant to participate in the Kyoto Protocol because they did not want to forego any future economic growth. While this argument is valid in the sense that developed countries have been using the environment to their advantage for the last 100 years, developing countries are in a very dangerous position if climate change does continue through the 21st century. Developing countries are more at risk of being ill-affected by climate change because of their geography, dependence on agriculture, and lack of resources and capital [Stern 2007]. Most developing countries are located in tropical areas, which will be the hardest hit by climate change and its corresponding extremes, like monsoons and very high temperatures. Their dependence on agriculture also puts them at risk because these extreme climate changes will inevitably affect crops, through floods or draughts. On top of all this, if these disasters hit a developing country, it will have difficulty coping with the consequences because of their lack of resources and capital. At this point within the debate, most countries agree that everyone should be involved in greenhouse gas emission reductions, but it is still unclear who should pay for it.

2.2 The Science and Uncertainties

Many scientists agree that a 350 parts per million (ppm) atmospheric concentrations of CO_2 is the safe upper limit [Ackerman et al. 2009]. This is the estimated highest point at which the world does not feel the effects of global warming. Up until about 200 years ago, before the Industrial Revolution, the stable atmospheric concentration of CO_2 was about 275 ppm. In 1990, the world emitted 22.5 million kilotons, passing the 350 ppm safe upper limit of atmospheric concentration of CO_2, reaching about 354 ppm. This is why most pollution reductions programs (especially the Kyoto Protocol) are relative to 1990s emission levels. The 1990 emission level is a desirable mile marker to measure reductions against because policy makers have an easy point of reference when justifying emission reductions in order to move back toward the safe zone of CO_2 concentrations. Telling countries that they must reduce emissions to their 1990 emission levels seems to be a more reasonable request than some astronomical abatement target, even though they are actually the same reduction amount. The world produced about 30.6 million kilotons of carbon dioxide in 2007 and currently has a concentration of 388 ppm CO_2, which is rising at about 2 ppm a year [350.org 2011]. This means that we must reduce annual CO_2 emissions by more than 8 million kilotons to reach 1990 emission levels, and more to stabilize and reduce the CO_2 atmospheric concentrations.

Uncertainty about global warming and CO_2 emission levels creates challenges when attempting to address the climate change issue because no one actually knows what the future will hold. Therefore, nations are unwilling to make costly decisions based on mere speculation. The first big issue with uncertainty is that the actual effects of global warming and to what degree they will occur are unknown. Even though scientists believe that 350 ppm is the safe upper limit of atmospheric CO_2 concentrations, there is no way to know that this is the correct level or even if the climate change can be stopped and controlled at this point. There is no proof that the environment will recover and return to the pre-1990s climate once CO_2 emissions are brought down to a reasonable level.

Even with the uncertainties of climate change and its mitigation, most world leaders and policy makers agree that climate change is a pressing world issue and that it is in everyone's best interest to find a solution. Currently, there are no global climate change agreements, but the United Nations participate in annual climate change summits in hopes of moving toward global mitigation. The next two sections look at the two opposing manners of imposing emission reductions–command-and-control standards and market-based emission regulation.

2.3 Command-and-Control Standards

Although the name "command-and-control standards" gives an automatic bad connotation, they are the most common form of emission reduction policies in both developed and developing countries. Yet command-and-control is an appropriate and telling name for the approach of these emission regulations. The "command" comes from a government or overarching organization that sets a particular standard or maximum level of pollution, and "control" comes from the monitoring and enforcement of the standard. Governments feel the need to regulate emissions since the environment is a public good, i.e., it must be protected from exploitation. There are three main types of command-and-control standards. First, emission standards are the most basic and logical emission regulation. They specify a certain maximum emission level, but do not specify how the standards are met. Second, technology standards specify certain greener technologies that firms must invest in. Last, ambient standards specify the maximum allowable concentration of pollutants.

Although command-and-control standards are extremely widespread and common, they have a number of disadvantages, which is why there has been a shift in the abatement discussion toward market-based emission regulation. Command-and-control standards are less cost-effective than market-based regulation because firms are not given alternative methods of achieving their abatement targets. For example, each power plant in a country must reduce their emissions by 5 percent, but some plants have a comparative advantage in reducing emissions. Suppose the coal-burning power plant can reduce ten percent of their emissions at a much lower cost than the hydroelectric plant can reduce their five percent. It is more cost-effective for the hydroelectric plant to pay for the additional five percent reduction at the coal-burning power plant than to reduce the five percent at their own plant. Also command-and-control standards give no incentive to reduce emissions past the emission standard. Returning to the above example, obviously the coal-burning power plant will only reduce its emissions by five percent and not the efficient ten percent. Thus, the emission standards must constantly be updated, which is difficult to keep up with because of the time consuming legislation that is required.

2.4 Market-based Emission Regulation

There are two main market-based emission regulation systems–cap-and-trade and carbon taxes. Both methods assign a price to greenhouse gas emissions, which is an additional cost on firms. This price provides firms with economic incentives to reduce their emission levels, which will inherently raise their operating costs. Market-based regulation addresses negative environmental externalities, correcting the market failure by incorporating these external costs of production and consumption into the market through taxes, or by creating a proxy market for emission rights. These methods can be used to not only internalize the costs of pollution, but to also reduce pollution if it is assigned a high enough cost. There has been a growing trend toward market-based regulations in the last thirty years because they are more cost-effective than command-and-control standards [Harrington and Morgenstern 2004].

The cap-and-trade system (or emission trading) does exactly what it sounds like. An IGO places an upper limit on greenhouse gas emissions. The property rights to this pollution are called emission permits, and are allocated to each country in some fair manner [Nordhaus 2008]. The IGO must use a justifiable method for fair distribution to encourage maximum participation in the cap-and-trade system. If countries do not believe the distribution is fair they will simply opt out of the trading scheme and emit as much as they want. Once all of the permits are allocated, countries may enter an emission permit trading market. Each country has the option of buying or selling their permits at any price. This price will inevitably converge to the price where all countries' marginal utilities are equal. A country that has high abatement costs will buy permits from countries that have low abatement costs. This means certain countries have a comparative advantage in pollution abatement. This comparative advantage leads to a cost-efficient market because the countries with the lowest abatement costs end up reducing their emissions.

Carbon taxes place a tax on carbon intensive technologies, including production, distribution and use of fossil fuels [Stern 2007]. The tax should equal marginal damage costs. The tax is in proportion to the technology's carbon content. The tax provides an incentive for decreasing carbon emissions while also raising tax revenue, which may be used to promote and research non-carbon technologies. Carbon taxes have a few advantages over cap-and-trade; they are flexible so that they are easily adjusted until a cost-effective tax rate is achieved, and they have lower compliance costs. Cap-and-trade systems have won the approval of politicians, but "many economists and consumers prefer carbon tax for its simplicity and impartiality" [Dowdey 2007]. Either way both carbon taxes and cap-and-trade accomplish the same goal of creating economic incentives to reduce emissions and promote alternative energy, like wind, solar, and geothermal.

The cap-and-trade method has been the most successful emission regulation method in the international realm. Even though there is no global climate agreement, there have been positive steps toward intergovernmental emission regulation, first with the Kyoto Protocol in 1997, and the European Union Emission Trading Scheme in 2005. These agreements are important precedents, models, and learning tools when developing a global climate agreement. The details of both will be discussed in the next section.

2.5 The Kyoto Protocol and European Union Emission Trading Scheme

The United Nations has developed a number of international agreements dealing with the mitigation of global carbon emissions over the last twenty years, but so far the most successful and well known is the Kyoto Protocol. Enacted in December 1997, the Protocol called for a reduction of six greenhouse gases–carbon dioxide, methane, nitrous oxide, sulfur hexafluoride, hydro-fluorocarbons and perfluorocarbons. Annex 1 (or developed) countries agreed to a collective 5.2% reduction of emissions from their 1990 levels. These reductions can be met through the flexibility mechanism outlined in the Protocol. Annex 1 countries may reduce either within their own border in a manner of their own discretion or internationally through emission trading, clean development mechanisms, and/or joint implementation [Schreuder 2009]. Clean development mechanisms are abatement projects that are hosted in non-Annex 1 (or developing) countries, where as joint implementation are hosted in other Annex 1 countries. The contributing country receives the emission reduction in return for paying for the hosted project in the host country. The idea is that countries with high marginal abatement costs can host projects in countries with lower abatement costs, taking advantage of comparative advantages between countries. These programs encourage cost-effective decisions. Non-Annex 1 countries are held to no obligation under the Kyoto Protocol, which is why the United States would not ratify it. Countries, like China and India, can continue polluting as much as they want.

Although the Protocol's implementation and enforcement has been insufficient, it is still an important development in the move toward a global system of carbon mitigation because it established the first global burden sharing on climate change. The largest of the Kyoto Protocol's faults is that the most powerful country in the world, the United States, didn't ratify it. The U.S. did not agree with the Protocol's choice to exclude growing developing countries, like China and India, from the required emission reductions. The U.S. believes that the world's next biggest emitters must also be subject to the same rules as industrialized countries. Any successful climate change mitigation method must have the support of both developing and developed countries, which will be a major challenge to overcome.

One of the most encouraging outcomes of the Protocol is the establishment of the European Union Emission Trading Scheme (EU ETS). European countries that ratified the Protocol organized their own emission trading system, which has effectively mitigated carbon emission from countries in the European Union. The EUETS is the largest international emission trading system in the world. The EU ETS functions as a cap-and-trade system, where there is a uniform price of carbon for greenhouse gas emissions from specific heavy industries, like energy generation, metal production, pulp and paper, and cement and bricks [Stern 2007]. The EU ETS has 25 members that each submit National Allocation Plans (NAPs) that help with the European Commission decision on national permit allocations. Within the EU ETS, firms both buy the permits and provide annual reports on emissions. The first phase lasted three years, 2005-2007, with the permit market worth around $115 billion. The EU ETS only regulates about 40% of the carbon emissions from the member countries, only focusing on emissions from heavy industries at this beginning stage of the trading scheme. Despite this low regulation coverage, the system is on the right track. The EU ETS has plans for increased reductions in additional industries and sectors in the future. It is important to note that almost all the countries that are reaching their Kyoto Protocol emission reduction goals are participating in the EU ETS. The system gives hope for more multi-national cooperation and a possible global emission trading system.

Chapter 3

Literature Review

This chapter reviews the existing literature concerning climate change policies, such as emission trading, carbon taxes, the Kyoto Protocol, and the EU ETS. The primary focus is on the differences in efficiency between the opposing policies, and any useful methods that we may want to incorporate into our model, like initial entitlements proportional to a 5.2% reduction of 1990 emission levels. The last section explores emission market simulations.

3.1 Emission Trading and Carbon Tax

Gerlagh, and van der Zwaan [2006] compile a survey of carbon emission abatement options–energy savings, less carbon-intensive resources, or carbon capture technology. They also analyze the following policy instruments–carbon taxes, fossil fuel taxes, renewable energy subsidies, portfolio standards for carbon energy, and portfolio standards for renewable energy–which are used to implement the different carbon abatement options. Unfortunately, the authors do not include emission trading within the comparison of abatement policy instruments. These instruments are compared in terms of cost, efficiency and impact on energy supply system. They find that carbon intensity portfolio standards are the most cost-efficient policy instrument because they use the tax revenues as subsidies for non-fossil fuel energy. The authors conclude that carbon capture and storage methods are undervalued by most emission abatement methods. This is the main cause of the efficiency difference between carbon and fossil fuel taxes. Unlike carbon intensity portfolio standards and carbon taxes, fossil fuel taxes do not take advantage of carbon capture and storage technologies because there are no economic incentives for the investment. But the authors note that it is possible to create economic incentives in parallel to a fossil fuel tax through separate treaties.

Kennedy and Laplante [1999] explore the technological incentives that carbon taxes and emission trading provide for regulated firms to adopt cleaner technologies by focusing on the time consistency issues, i.e. policy makers not being able to find appropriate abatement targets to achieve efficiency. Although the authors note that the results do not strongly favor one system over the other, they find that the rational expectations equilibrium with emission taxes gives excessive incentives, where as emission trading gives weak incentives. How efficient these systems are depends on whether the function of environmental damage is linear or strictly convex. Their results suggest that if the damage is linear then there must be either universal adoption of new technologies or universal retention of old technologies. On the other hand, if damage is strictly convex then there must be strict partial adoption of the new technology. The authors finally conclude that the carbon tax has a possible simplicity advantage over trading because they do not have to continually adjust the supply of permits, which can suffer from legislative delays. These points are important because it speaks to how countries must implement their carbon abatement measures. Policy makers must be aware of possible implementation delays, which throw off efficiency. A model that is more flexible with its standards and methods may possibly achieve efficiency sooner.

Aldy, Ley, and Parry [2008] also discuss the differences between emission taxes and trading, but they focus more on the design aspect of the systems. The authors assert that for an international emissions agreement to be considered "successful" it should be cost-effective, distributionally equal, broadly participated in, generally agreed upon, well regulated, and have reasonable domestic abatement capabilities. It appears that although both trading and taxes may meet the first three criteria, a tax-based system may be much simpler to implement because it can be opposed upstream, i.e. on fuel producers. Upstream implementation leads to simplicity because the regulators would only have to deal with 2,000-3,000 fossil fuel producing firms versus all firms that use fossil fuels. It is important to note that the taxes are eventually paid by the consumer, but in a indirect manner. They also find that taxes are better than emission trading schemes when comparing them on the grounds of uncertainty, and fiscal and distributional issues, but it also depends on the tax policies and how the revenues are used. On the other hand, emission taxes might be a problem in the international setting because they may offset changes created by other energy policies. This so-called "tax-interaction effect" drives product prices upward and therefore depresses consumers' purchasing power, which inevitably leads to further depression of the labor supply and capital accumulation. Despite these issues, the authors still argue that a carbon-based tax would have design advantages over emission trading, especially on an international level because they are more efficient at promoting broad participation, especially in developing countries.

3.2 The Kyoto Scheme

Caplan, Cornes, and Silva [2003] investigate the efficiency of the Kyoto Protocol and its programs if a major emitter, like the United States, does not participate. They find that if any major emitter fails to participate the Kyoto Protocol becomes inefficient because the country is not held accountable for its carbon emissions and therefore imposes negative effects on the rest of the world. It is also inefficient because the major emitter is most likely also a major capital holder, who is not involved in the resource and permit transfers, so others fall short of their maximum trade gains. The authors explore a new situation called 'the Ideal Kyoto Protocol,' which imposes emission trading in the form of redistributive transfers and global participation. Therefore, every region participates in the emission trading scheme and the redistributive transfer mechanism should operate only after governments have made their policy commitments on how to control their carbon emissions. They find that the Ideal Kyoto Protocol achieves a Pareto efficient equilibrium on the global market. The comparison between the Kyoto Protocol with and without the U.S. in this article shows it is necessary for all countries to participate in trading schemes and emission abatement efforts.

Bhatti, Lindskow, and Pedersen [2010] examine the distribution of burden sharing for abatement costs under the Kyoto Protocol empirically with an OLS regression. They are trying to figure out who is actually paying for the pollution abatement under the Protocol. They found that countries were compensated for early action. Wealthy countries (who have the highest emissions), those with high projected growth rates, and potential EU members have the strictest abatement targets. This article does not addresses the question of what is actually "fair," but it enlightens us to the ways in which the abatement costs were divided among countries. The authors also discuss the efficiency hypothesis, which argues that the lower the energy efficiency of a country, the stricter the reduction requirements are, but they find that it has no supporting evidence. This means that pollution allowances were not distributed based on countries' marginal abatement cost curve. One would hope that the market would fix this issue through emission trading because these inefficient countries would have a comparative advantage in emission reductions. But the point that the authors make is that the initial negotiations and emission allowances influence who actually pays for the emission abatement.

Martin [1999] explores one of the Kyoto Protocol's flexibility mechanisms, called joint implementation. This program is a very important aspect of the Kyoto Protocol because the Protocol's emission trading scheme only involves Annex 1 countries. Since the emission trading scheme is only between developed countries, it is not taking advantage of the lowest possible marginal abatement costs, which occur mostly within developing countries because they have not yet implemented basic abatement technology and equipment that has been available and required within developed countries for more than a decade. The joint implementation of energy projects have potentially important second-round impacts because in order to reduce emissions in developing countries more technologically efficient capital is installed. These impacts often lower prices which leads to an increase in quantity demanded and further creates substitution effects. Since these new technologies are the least emission-intensive, the price effects are most likely to be favorable.

Joint implementation is a possible alternative to emission trading in the sense that it is based on bilateral abatement and transfer payment agreements, rather than a global market. But one would hope that such a system would lead to a competitive market outcome, which would in the end be quite similar to emission trading. The difference is that the Kyoto Protocol used joint implementation instead of forcing developing countries to be involved in the emission trading scheme because they wanted to give developing countries the choice to abate pollution. This further delves into the policy debate about what is actually fair for developing countries because they did not cause the majority of the pollution. Martin [1999] finds that joint implementations is efficient and appears to have a good impact on emission abatement. Like carbon capture and storage emission reduction methods, the question is if joint implementation can occur on a large enough scale to create large carbon emission reductions because it is an optional program. This is one of the main reasons why developing countries should participate in the global emission schemes. Even though joint implementation is an efficient method of reducing emissions, its optional nature does not force the developing countries to participate. Thus giving countries a choice decreases the efficiency of the system as compared to if they participated in the emission trading scheme with the Annex 1 countries. This was the U.S.'s major criticism with Kyoto; it did not hold developing countries responsible for their pollution as it did developed countries.

Hagem [2009] discusses the last aspect of the Kyoto Protocol's scheme for pollution abatement, the Clean Development Mechanisms (CDMs), which is another flexibility mechanism. Hagem mainly compares the efficiency of emission trading versus CDMs within developing countries with a focus on the incentives to invest in new technologies for pollution abatement and their actual gains from investment. Thus, this article does not focus on the CDMs effect on minimizing developed countries costs, which is the other aspect of the CDM program. Hagem finds that the gains from incentives come from developing countries facing either competitive or noncompetitive output markets. Within a perfectly competitive market, gains from investment under the CDMs are larger than those under an emission trading scheme. The reverse is true for an imperfectly competitive market, so gains from investment under an emission trading scheme are larger than those under a CDM regime.

3.3 The European Union Emission Trading Scheme

In 2007, the EU began reviewing the EU ETS's market functions in order to find areas for improvement. Daskalakis and Markellos [2008] took on this challenge of assessing the efficiency of the EU ETS for CO_2 emissions during its first two years of operation. They focus on the weak form of efficiency, in which current prices should reflect all the information in previous prices, i.e. any market forecasts are developed from the most recent price. They also specifically look at the three largest regional emission exchange markets–Powernext (France), European Climate Exchange (Netherlands), and Nord Pool (Scandinavia). The results suggest that trade gains are serially predictable, which can be exploited to produce substantial risk-adjusted profits. Although markets are not currently consistent with the weak form of efficiency, the authors claim this may be due to the immaturity of the scheme and the restrictions–short-selling and "banking" emissions permits. Unfortunately, the authors make a very good point for the inefficiencies of the EU ETS, but do not offer any further suggestions of how the EU should rectify these issues.

Ehrhart, Hoppe, and Loschel [2008] explore the loopholes in emission trading systems, specifically the EU ETS, and how they affect the efficiency of the program. The loopholes stem from the misuse of trading institutions, like pooling and project-based mechanisms. They are called loopholes because they are technically legal but affect the product market in an unintended manner. In order to analyze the affects of loopholes on the EU ETS they use two game theoretical models that feature oligopolistic firms that face perfect competition. In these models the firms are always price takers in the allowance market. The first model is of duopolistic firms that set their output levels and then their cost-minimizing abatement levels. It ends up that comparative advantage leads to firm profits when allowance prices rise. This model ends up producing a market with a lower production level, high market prices and higher profits. Thus, the overall social welfare within this system decreases. In the second model, the duopolistic firms first cooperatively determine their number of permits and emission levels while simultaneous choosing their product output levels. The firms within the second model enjoy comparatively higher profits. This leads to fewer firms buying permits and abating more than the cost-minimizing model calls for. Thus, the authors find that loopholes within the system actually foster tacit collusion, which creates oligopolistic markets. This creates profits for the firm and has negative social welfare effects. This is a warning for what policy makers should watch out for within emission trading legislation that could lead to the restriction of competition and market inefficiencies. The authors note that CDMs are particularly susceptible to these loopholes.

3.4 Emission Market Simulations

Nordhaus [1992] developed the dynamic integrated climate-economy model (DICE), which is used to simulate alternative abatement methods. The DICE model is important because it investigates the implications of economic growth on the environment. Most simulations of carbon markets do not account for the region's economic growth and the pressures it would put on the abatement system. Developing countries are afraid of agreeing to set emission targets because they do not want CO_2 emission reductions to constrict their rate of growth. The DICE model uses economic growth theory combined with the dynamics of emissions creating a closed-loop interaction between the climate and economy. Thus, the model accounts for the economy through the initial stock of capital, labor and technology and gets the output from a Cobb-Douglas production function. Further, capital accumulation is determined by consumption over time, where as population growth and technological change are both exogenous. The climate side of the model is determined by emission, concentrations, and climate equations, a damage relationship, and a marginal abatement cost curve.

Nordhaus [1992] also investigates the efficiency differences between five alternative abatement policies–no controls, optimal policy, ten-year delay of optimal policy, twenty percent emission reduction from 1990 levels and geo-engineering. Nordhaus finds that a "modest" carbon tax (i.e. optimal policy) would be efficient. On the other hand, a twenty percent emission reduction from 1990 levels would be extremely costly and unrealistic.

Carlen [2003] explores how emission trading will be affected by market power, and whether large countries would be able to manipulate emission prices. The article further discusses the efficiency of emission trading in a transparent market where traders have accurate information about net demand and completes an emission trading simulation on a subset of 12 countries in order to observe the outcome. In the market simulations, Carlen uses a dual action (DA) market in which trade is sequential so sellers and buyers make bids and ask or accept them from other traders. Thus, the market may have some price discrimination. Through the simulation, Carlen found that trade levels and prices actually converge on to competitive levels, and that a large trader does not create substantial inefficiencies. In the majority of the trading periods, the traders achieve more than 95% of the maximum trade gains.

Similarly, Stranlund, Murphy, and Spraggon [2008] examine how imperfect enforcement affects the efficiency of emission trading markets. They ran laboratory experiments to simulate risk-neutral profit maximizing firms, who are not in full compliance. The experiment was conducted on a volunteer student population at the University of Massachusetts, Amherst. The subjects conducted emission reductions and transfers on a DA market, but were allowed to have different amounts of pollution and pollution permits, i.e. imperfect enforcement. The results were then compared to outcome predictions based on emission trading theory. They found that their imperfectly enforced emission trading markets were “reasonably” efficient, but expected penalties were lower than predicted when they should have been high, but about on par when they should have been low. Although the market has reasonable efficiency when compared to a perfectly enforced emission trading market, its penalties and violations were lower. Even though the simulation that we run on Helm’s emission trading scheme will be perfectly enforced, it is important to keep in mind how the results could be affected if countries exceed their emission allowances. It brings up the question of penalties and how violations should be handled in the international system.

Chapter 4

Model

This chapter introduces fair division problems and the equity criteria required to have a fair division solution. We examine the fair division problem of carbon emission permits, and Helm's bounded Walrasian solution. We prove that the bounded Walrasian solution guarentees four fair division equity criteria–Pareto efficiency, individual rationality, envy-freeness with respect to a compensation rule, and the stand-alone upper bound.

4.1 Fair Division Problems and Solutions

4.1.1 The Problem

A fair division problem is a homogeneous, scarce, common resource, $\omega \in R_+$, that must be divided among n agents, $I = \{1,2,...,n\}$. The agents have particular preferences for the resource, U_i, which are continuous utility functions. Each agent has a stand-alone utility amount, $U_i(\omega)$, that is associated with the utility the agent would receive if he were to consume the entire common resource. Each agent starts with an initial nonnegative entitlement allocation of the common resource, ω_i, such that

$\sum_{i \in I} \omega_i \leq \omega$. This fair division problem is defined as $E \equiv \{U_i, \omega_i\}_{i \in I}$. The point of fair division is to divide the common resource in a manner that the recipients believe is fair.

The fair division problem between two players has existed for centuries dating back to its reference in the Old Testament.

Example 4.1.1.1. (The Story of Abram and his nephew, Lot).

> 6 And the land could not sustain them while dwelling together, for their possessions were so great that they were not able to remain together.
> 7 And there was strife between the herdsmen of Abram's livestock and the herdsmen of Lot's livestock ...

In this fair division problem, ω is the land, the agents are Abram and Lot, and ω_{Abram} and ω_{Lot} are both zero because neither have any of the land at this point. Note that this extremely simple problem does not specify preferences, U_i.

4.1.2 The Solution

A fair division solution is a particular way of dividing the common good, ω, so each agent receives a nonnegative allocation, x_i, such that $\sum_{i \in I} x_i \leq \omega$. Thus, define the solution to E as $F \equiv \{x_i\}_{i \in I}$.

Example 4.1.2.1. (Returning to the Story of Abram and his nephew, Lot).

> 8 So Abram said to Lot, "Please let there be no strife between you and me, nor between my herdsmen and your herdsmen, for we are brothers.
> 9 "Is not the whole land before you? Please separate from me; if to the left, then I will go to the right; or if to the right, then I will go to the left."
> (Gensis 13: 6-9)

Lot travelled east, choosing the valley of Jordan, and Abram received the land of Canaan.

In Abram and Lot's fair division solution, x_{Abram} is the land to the left and x_{Lot} is the land to the right. Although the agent's preferences were not specified in the fair division problem, we can assume that both agents are satisfied with their final allocation because they agreed to the division. If either agent was unhappy with his portion, he would have continued arguing for more land.

4.1.3 The Three Equity Criteria of Fair Division

A fair division solution may be subject to many equity criteria, but the three we discuss here are Pareto efficiency, individual rationality, and envy-freeness.

The first and most basic fair division equity criterion is individual rationality. In layman's terms, an individually rational solution requires that no agent agrees to an allocation that makes him worse off than his starting point. This restriction appears to be common sense, but it is important to include because within a fair division there should not be any agents who are net losers.

Definition 4.1.3.1. An allocation $(x_i)_{i \in I}$ is individually rational if

$$U_i(x_i) \geq U_i(\omega_i) \text{ for all } i \in I.$$

To better understand what makes an allocation individually rational, we will once again refer to the discussion of Example 4.1.2.1. In order for Abram's final allocation to be individually rational, he will only participate in a trade if his utility remains the same or increases. To illustrate Abram making a rational decision we will present an example where he will not want to participate in the trade.

Example 4.1.3.2. (An Individually Rational Decision Continuation of Example 4.1.2.1). Suppose Lot's valley of Jordan has two wells in the north and Abram's land of Canaan has one well in the south as well as one in the north, all of which are close to the division line between Lot's and Abram's lands. Lot proposes a trade of one of his wells in the north for Abram's only well in the south. It is easy to see that although Lot will be better off because because of trade for his herdsmen do not have to travel as far for water, Abram will be worse off because both his wells will be in the north. Abram is an individually rational agent if he does not trade his southern well for a northern one. He makes this decision because it will make life much more difficult for his herdsmen who will not be as productive, and therefore lower the amount of utility he receives from his lands.

The individual rationality equity criterion places a lower bound on each agent's utility, so that each participating agent to guarenteed at least the utility from his initial allocation.

The second fair division equity criterion is Pareto efficiency. In layman's terms, a Pareto efficient solution requires that any change to the solution $\{(x_i)\}_{i \in I}$ that makes one agent better off must make another agent worse off.

Definition 4.1.3.3. A final allocation $(x_i)_{i \in I}$ is Pareto efficient if there does not exist a feasible allocation $(x_i^*)_{i \in I}$ such that

$$U_i(x_i^*) > U_i(x_i) \quad \text{for some} \quad i \in I \quad \text{and}$$
$$U_i(x_i^*) \geq U_i(x_i) \quad \text{for all} \quad i \in I.$$

To better understand what is a Pareto efficient allocation and what is not, we will return to the dicussion of Example 4.1.2.1. The land division that was decided in Example 4.1.2.1 is Pareto efficient if there does not exist a trade of lands such at least one man is made happier without making the other worse off. Perhaps the easiest way to understand is an example where Pareto efficiency is not satisfied.

Example 4.1.3.4. (A Non-Pareto Efficient Continuation of Example 4.1.2.1).
Suppose Lot's valley of Jordan has two wells in the north and Abram's land of Canaan has two wells in the south, all of which are close to the division line between Lot's and Abram's lands. If Lot trades one of his wells in the north for one of Abram's in the south, both men will be better off because they now both have wells in the north and south, so their herdsmen do not have to travel as far for water. Thus the initial division in Example 4.1.2.1 was not Pareto efficient.

On the other hand, if the initial division in Example 4.1.2.1 were Pareto efficient, at least one man would not want to trade because he would be made worse off. Suppose one has already developed an expansive system of aquifers; then he would not be willing to trade his one of his two wells because both are being used to water all of lands.

The third equity criterion is envy-freeness.

Definition 4.1.3.5. (The Usual Definition of Envy-freeness.)
An allocation is $(x_i)_{i \in I}$ is envy-free if for all pairs of agents $i, j \in I$,

$$U_i(x_i) \geq U_i(x_j).$$

In layman's terms, envy-freeness is the property that no agent prefers another agent's allocation to his own, i.e. Agent i does not prefer Agent j's allocation, x_j, over his own, x_i.

To better understand what envy-freeness means, we will return to the discussion of Abram and Lot. In order for both Lot and Abram to be envy-free, neither should want what the other has. To illustrate this, we will return to Example 4.1.3.2 where one agent, Lot, envies the other, Abram.

Example 4.1.3.6. (An Envy-freeness Violation Continuation of Example 4.1.2.1). Consider the set up in Example 4.1.3.2, where Lot has two wells in the north and Abram has one well in the south and one in the north. Since the farthest Abram's herdsmen must travel for water is half way across his lands, where as Lot's must travel all the way across his lands to reach a well, both men would prefer Abram's land over Lot's. Lot would much rather have Abram's lands than his own because the herdsmen would be much more productive if they were not always walking to reach water. Thus, Lot envies Abram.

Remark 4.1.3.7. If the individual rationality and envy-freeness criteria are satisfied, it is more likely that all agents would be willing to participate in the fair division solution, instead of wanting to opt out and arguing for an even larger portion of the common resource.

4.2 Fair Division Algorithms

Mathematicians have developed a number of fair division algorithms, which guarentee a solution satisfies particular equity criteria. In this section, we will discuss the two most basic and well-known fair division algorithms and their associated solutions.

4.2.1 The Divide-and-Choose Method.

The most basic fair division problem is between two agents. When two agents must "fairly" divide a common resource (e.g. a piece of cake) amongst themselves, the simplest solution is to cut the piece of cake in half. But this is only a fair division solution if both participants have the same preferences. If they have differing preferences, e.g. Agent 1 likes the corner piece because it has more frosting and Agent 2 likes the inside piece because it has the most chocolate, then one participant may want the other's pieces of cake compared to his own when the whole cake is simply halved. Thus the agents employ the Divide-and-Choose Method to find a fair division of the cake.

Algorithm 4.2.1.1. (The Divide-and-Choose Method).
Step 1: One agent (either one) cuts the piece of cake where he believes the two pieces are equal.
Step 2: The other agent chooses whichever piece he believes is equal or greater than the other.
Therefore both participants are receiving a piece of the cake that each believes is equal or greater than the other's [Robertson and Webb 1998].

Returning to 4.1.2.1, the Divide-and-Choose Method is exactly the setup that Abram proposes. He picked a spot on the land that he would not mind being on the left or the right side of, and then he let Lot choose whichever side he wanted. Both men were content with their final piece of land, knowing they thought their pieces were just as good as or better than the other's.

It is important to note that the Divide-and-Choose Method for two agents results in an envy-free and proportional division. Proportionality refers to a division outcome where each agent receives what he believes to be at least $1/n^{\text{th}}$ of the cake. On the other hand, envy-freeness refers to a division outcome where no agent wants any other's allocation compared to his own, i.e., each agent believes his allocation is preferable to the other agents' allocations.

This solution to the two agent fair division problem is quite simple and easily understood algorithm. Only two decisions are being made: first the cut, and then the choice.

4.2.2 The Moving Knife Algorithm.

The jump to the three agent problem is much more difficult than the two agent problem. The solution is not quite as intuitive as the two agent problem. Although there are many different algorithms to find a fair division between three agents, we are presenting the Moving Knife Algorithm because it can be generalized for n agents.

Algorithm 4.2.2.1. (The Moving Knife Algorithm for Three Agents).
Step 1: A knife continuously moves over a piece of cake from left to right. Any agent who thinks the piece to the left of the knife is 1/3 of the entire cake yells "Cut." The knife cuts the cake at this place and this agent receives the cut piece of cake and drops out of the division algorithm.
Step 2: The next agent who thinks the piece to the left of the knife is 1/3 of the entire cake yells "Cut." Again this knife cuts the cake at this place and the agent receives his cut piece of cake and drops out of the division algorithm.
Step 3: The final agent receives the remaining pieces of cake [Robertson and Webb 1998].

The Moving Knife Algorithm can be extended to n agents by simply changing 1/3 of the entire cake to $1/n$ and iterating Step 2 until there is only one agent left, who moves on to Step 3 and receives the remaining piece of cake. Although this method achieves proportionality, it does not result in an envy-free division. Even though each agent is choosing what he believes is a "fair" $1/n$ piece of cake, the agent who first yells cut will always have the smallest piece of cake, and may therefore envy those who cut pieces that are larger afterward.

Again, consider the story of Abram and Lot (Example 4.1.2.1) and what would occur if Abram and Lot employed the Moving Knife Algorithm, instead of the divide-and-choose method.

Example 4.2.2.2. (Abram and Lot using the Moving Knife Algorithm).
To "fairly" divide the land, Abram and Lot begin walking westward from the eastern boundary of the land. At some point along the way Abram decides that they have covered half of the land and yells "Cut!" At this point, Abram and Lot put up a fence perpendicular to the northern boundary that cuts the land in two. Abram receives the land of Canaan that they just traversed, and Lot receives everything to the west of the fence, the valley of Jordan.

4.3 The Fair Division of Emission Permits and Helm's Model

The fair division problem that we are looking at in this paper is the fair division of emission permits in a carbon emission trading scheme, i.e. cap-and-trade. As described earlier, cap-and-trade puts a ceiling on carbon emissions, and creates a market price for emission permits, which countries or firms may buy in a free market.

4.3.1 The Fair Division Problem of Emission Permits.

In the fair division problem of emission permits, the common resource, ω, is the capped number of availible emission permits. The agents are countries participating in the emission trading scheme. Each country receives a nonnegative entitlement allocation of emission permits, ω_i. The initial permit entitlements are based on a predetermined and justifiable entitlement measure. For example, each country receives a certain number of permits per capita or proportional to their 1990 emission levels. After the emissions trading, each agent has a final allocation bundle (x_i, t_i), where $x_i \in R_+$ is the final allocation of emission permits and t_i is the transfer payment. A transfer payment is a monetary value that represents either a revenue for selling excess permits or expenditure for buying additional permits. In order to find the equilibrium trading point, each agent has unique preferences that are characterized by a continuous monotonic and quasilinear utility function, $U(x_i, t_i) = v_i(x_i) + t_i$. This fair division problem is defined as $E \equiv \{U_i, \omega_i\}_{i \in I}$. Thus, the solution to E is defined as $F \equiv \{x_i, t_i\}_{i \in I}$. It is important to note the restrictions on F that $\sum_{i \in I} t_i = 0$ and $\sum_{i \in I} x_i \leqslant \omega$. The first restriction reflects that every transaction involving a monetary transfer is recorded twice as a negative to the buyer and a positive to the seller. The latter imposes that the solution $x_{i\, i \in I}$ is constrained by the common resource, i.e. the sum of all the agents final allocations of emission permits may not be larger than the common resource, ω.

The solution F to the fair division of emission permits may be subject to the three equity criteria discussed in Section 4.1.3.

Definition 4.3.1.1. An allocation $(x_i, t_i)_{i \in I}$ is individually rational for E if

$$v_i(x_i) + t_i \geq v_i(\omega_i) \text{ for all } i \in I.$$

Remark 4.3.1.2. It is important to note that the definition of individual rationality within a trading scheme is recursive. This means if the agents in I were to participate in an additional trading scheme, E', after the initial trade, E, then an allocation $(x_i', t_i')_{i \in I}$ is individually rational for E' if

$$v_i(x_i') + t_i' \geq v_i(x_i) + t_i \text{ for all } i \in I.$$

Definition 4.3.1.3. An allocation $(x_i, t_i)_{i \in I}$ is Pareto efficient if there does not exist a feasible allocation $(x_i^*, t_i^*)_{i \in I}$ such that

$$v_i(x_i^*) + t_i^* > v_i(x_i) + t_i \text{ for some } i \in I \text{ and}$$

$$v_i(x_i^*) + t_i^* \geq v_i(x_i) + t_i \text{ for all } i \in I.$$

Definition 4.3.1.4. An allocation is $(x_i, t_i)_{i \in I}$ is envy-free if for all pairs of agents $i, j \in I$,

$$v_i(x_i) + t_i \geq v_i(x_j) + t_j.$$

There is an interesting proposition that we can derive directly from Pareto efficiency within a trading scheme.

Since agents' utility functions, $U(x_i, t_i) = v_i(x_i) = t_i$ are quasilinear, transfer payments are valued in the same manner as the utility of emission permits. Assuming that lump-sum transfer payments are feasibly, the Pareto efficiency equity criterion imposes that a solution $(x_i, t_i)_{i \in I}$ is Pareto efficient if there exists no other feasible allocation $(x_i^*, t_i^*)_{i \in I}$ such that

$$\sum_{i \in I} (v_i(x_i^*)) + t_i^* > \sum_{i \in I} (v_i(x_i) + t_i). \tag{4.3.1.5}$$

Lump-sum transfers can be used as a method to redistribute utility between agents in the form of money without losing efficiency.

Proposition 4.3.1.6. *Given the assumption that lump-sum transfers are feasible, an allocation $(x_i, t_i)_{i \in I}$ is Pareto efficient if and only if it maximizes the sum of utility across agents.*

To prove Proposition 4.3.1.6, we will actually prove the negation, i.e. an allocation is not Pareto efficient $\Leftrightarrow$ the sum of utility across agents is not maximized.

Proof. (⇒) Suppose $(x_i, t_i)_{i \in I}$ is not Pareto efficient. Then there exists an allocation $(x'_i, t'_i)_{i \in I}$ such that

$$v_i(x'_i) + t'_i > v_j(x_j) + t_j \text{ for some } j \in I \tag{4.3.1.7}$$

$$v_i(x'_i) + t'_i \geqslant v_i(x_i) + t_i \quad \text{for all} \quad i \in I \tag{4.3.1.8}$$

We must show that the sum of utility is not maximized across agents, i.e., that

$$\sum_{i \in I} v_i(x'_i) + t'_i > \sum_{i \in I} v_i(x_i) + t_i.$$

From Equation (4.3.1.8), we have

$$\sum_{i \in I} v_i(x'_i) + t'_i \geq \sum_{i \in I} v_i(x_i) + t_i.$$

But from Equation (4.3.1.7), we know

$$\sum_{i \in I} v_i(x'_i) + t'_i > \sum_{i \in I \text{ for } i \neq j} [v_i(x_i) + t_i] + v_j(x_j) + t_j.$$

i.e. $(x_i, t_i)_{i \in I}$ does not maximize the sum of utility across agents.

(⇐) Suppose $(x_i, t_i)_{i \in I}$ does not maximize the sum of utility. Then there exists $(x'_i, t'_i)_{i \in I}$ such that

$$\sum_{i \in I} v_i(x'_i) + t'_i > \sum_{i \in I} v_i(x_i) + t_i.$$

We must show $(x_i, t_i)_{i \in I}$ is not Pareto efficient. We must produce an allocation

$(\hat{x}_i, \hat{t}_i)_{i \in I}$ such that

(a) $v_i(\hat{x}_i) + \hat{t}_i > v_i(x_i) + t_i$ for some $i \in I$ and

(b) $v_i(\hat{x}_i) + \hat{t}_i \geq v_i(x_i) + t_i$ for all $i \in I$.

Let $\mathscr{W} = \{w \in I | v_w(x'_w) + t'_w \geq v_w(x_w) + t_w\}$ and $\mathscr{L} = \{l \in I | v_l(x'_l) + t'_l < v_l(x_l) + t_l\}$. If $\mathscr{L} = \emptyset$, we are done because all agents are in $\mathscr{W}$, i.e. $v_w(x'_w) + t'_w \geq v_w(x_w) + t_w$, which satisfies (b), and as $\sum_{i \in I} v_i(x'_i) + t'_i > \sum_{i \in I} v_i(x_i) + t_i$, we know $v_i(\hat{x}_i) + \hat{t}_i > v_i(x_i) + t_i$ for some $i \in I$, which satisfies (a).

Assume $\mathscr{L}$ is nonempty. Define

$$W = \sum_{w \in \mathscr{W}} \left[v_w(x'_w) + t'_w - v_w(x_w) - t_w \right] > 0 \quad \text{and}$$
$$L = \sum_{l \in \mathscr{L}} \left[v_l(x_l) + t_l - v_l(x'_l) - t'_l \right] > 0.$$

W is all the utility gained because of the trade, where as L is all the utility lost because of the trade. Further, we know $W - L > 0$, which is the net gain in total utility in moving from $(x_i, t_i)_{i \in I}$ to $(x'_i, t'_i)_{i \in I}$. We will redistribute among utility agents through the transfer payments, so that all agents are at least as well off as with the initial allocation. But we will also make sure that the new allocation produced by the redistribution has the same utility as $(x'_i, t'_i)_{i \in I}$.

Define r, such that $0 \leq r = \frac{L}{W} < 1$. Let an additional transfer payment for the winners be $\tau_w = -r[v_w(x'_w) + t'_w - v_w(x_w) - t_w]$. Let an additional transfer payment for the losers be $\tau_l = [v_l(x_l) + t_l - v_l(x'_l) - t'_l]$. Consider the allocation $(x'_i, t'_i + \tau_i)_{i \in I}$.

Now, we must prove the following four claims:

Claim 4.3.1.9. We must show that $(x'_i, t'_i + \tau_i)_{i\in I}$ is a feasible allocation, i.e.

$$\sum_{i\in I}(t'_i + \tau_i) = 0.$$

We know $\sum_{i\in I} t'_i = 0$ as $(x'_i, t'_i)_{i\in I}$ is a feasible allocation. Thus we must show

$$\sum_{i\in I}\tau_i = 0.$$

We have

$$\begin{aligned}
\sum_{i\in I}\tau_i &= \sum_{w\in\mathscr{W}}\tau_w + \sum_{l\in\mathscr{L}}\tau_l \\
&= \sum_{w\in\mathscr{W}} -r[v_w(x'_w) + t'_w - v_w(x_w) - t_w] + \sum_{l\in\mathscr{L}}[v_l(x_l) + t_l - v_l(x'_l) - t'_l] \\
&= -r\sum_{w\in\mathscr{W}}[v_w(x'_w) + t'_w - v_w(x_w) - t_w] + \sum_{l\in\mathscr{L}}[v_l(x_l) + t_l - v_l(x'_l) - t'_l] \\
&= -rW + L \\
&= -\frac{L}{W}(W) + L \\
&= 0
\end{aligned}$$

Claim 4.3.1.10. We must show if $w \in \mathscr{W}$, then $v_w(x'_w) + t'_w + \tau_w > v_w(x_w) + t_w$. Let

$w \in \mathscr{W}$ be arbitrary. We have

$$\begin{aligned} v_w(x'_w) + t'_w + \tau_w - v_w(x_w) - t_w &= v_w(x'_w) + t'_w - r\left[v_w(x'_w) + t'_w - v_w(x_w) - t_w\right] - v_w(x_w) - t_w \\ &= v_w(x'_w) + t'_w - r\,v_w(x'_w) - r\,t'_w + r\,v_w(x_w) + r\,t_w - v_w(x_w) - t_w \\ &= v_w(x'_w) - r(v_w(x'_w)) + t'_w - r(t'_w) - v_w(x_w) + r(v_w(x_w)) - t_w + r(t_w) \\ &= (1-r)\,v_w(x'_w) + (1-r)\,t'_w - (1-r)(v_w(x_w) - (1-r)\,t_w) \\ &= (1-r)\left[v_w(x'_w) + t'_w - v_w(x_w) - t_w)\right] \\ &> 0 \end{aligned}$$

as $0 < r < 1$ and $v_w(x'_w) + t'_w - v_w(x_w) - t_w) > 0$ by definition of $w \in \mathscr{W}$.

Thus, $v_w(x'_w) + t'_w + \tau_w \geq v_w(x_w) + t_w$ for all $w \in \mathscr{W}$.

Claim 4.3.1.11. We must show if $l \in \mathscr{L}$, then $v_l(x'_l) + t'_l + \tau_l \geq v_l(x_l) + t_l$. Let $l \in \mathscr{L}$ be arbitrary. We have

$$\begin{aligned} v_l(x'_l) + t'_l + \tau_l &= v_l(x'_l) + t'_l + [v_l(x_l) + t_l - v_l(x'_l) - t'_l] \\ &= v_l(x_l) + t_l \quad \text{for all} \quad l \in \mathscr{L}. \end{aligned}$$

Claim 4.3.1.12. Let $\frac{L}{W} < r < 1$. Define $\tau_w = -r[v_w(x'_w) + t'_w - v_w(x_w) - t_w]$ and $\tau_l = v_l(x_l) + t_l - v_l(x'_l) - t'_l + \frac{1}{|\mathscr{L}|}(rW - L)$. Then we must show $v_i(x'_i) + t'_i + \tau_i \geq v_i(x_i) + t_i$ for all $i \in I$. We know $v_w(x'_w) + t'_w + \tau_w \geq v_w(x_w) + t_w$ for all $w \in \mathscr{W}$ by Claim 4.3.1.10.

Let $l \in \mathscr{L}$ be arbitrary. We have

$$\begin{aligned} v_l(x'_l) + t'_l + \tau_l - v_l(x_l) - t_l &= v_l(x'_l) + t'_l + \left[v_l(x_l) + t_l - v_l(x'_l) - t'_l + \frac{1}{|\mathscr{L}|}(rW - L) \right] - v_l(x_l) - t_l \\ &= \frac{1}{|\mathscr{L}|}(rW - L) \\ &> \frac{1}{|\mathscr{L}|}(\frac{L}{W}W - L) \\ &> 0 \quad \text{for all} \quad l \in \mathscr{L} \quad \text{because we assumed} \quad \mathscr{L} \quad \text{is nonempty.} \end{aligned}$$

Thus, $v_i(x'_i) + t'_i + \tau_i \geq v_i(x_i) + t_i$ for all $i \in I$.

□

4.3.2 The Walrasian Equilibrium

The Walrasian equilibrium is a vector consisting of the commodity price and an allocation of the resource to agents in a competitive market such that each agent maximizes his own utility function. But this maximization is contrained because the equilibrium must ‘clear the market’, which means equating aggregate supply and demand for the traded commodity. Note that since it is a competitive market, no agent has the power or influence to affect the market price. The Walrasian equilibrium is the market equilibruim used in cap-and-trade schemes.

Within an emission trading scheme, a Walrasian equilibrium is found by agents maximizing their utility according to the price. The price is found where marginal utility is equal across all agents. The maximizing transfer payment is found my the maximization of the utility function where their revenue and expenditure for the emission permits are calculated by multiplying the change in an agent’s permit quantity by the price, $t_i = p(\omega_i - x_i)$.

Definition 4.3.2.1. Let the allocation $(x_i)_{i \in I}$ and the price $p \in R_+$ constitute a Walrasian equilibrium for E. Then for all pairs of agents $i, j \in I$:

$$v_i(x_i) + p(\omega_i - x_i) \geq v_i(x_j) + p(\omega_i - x_j).$$

This means that if $(x_i)_{i \in I}$ is a Walrasian equilibrium, then agent i's utility is maximized across all agents, i.e. agent i has chosen x_i rather than x_j because he will receive higher utility for x_i than for x_j.

The the resource allocation from a Walrasian equilibrium is a useful solution to the fair division problem for emission permits because it has a number of well known economic conclusions. The most important is that the Walrasian allocations are both Pareto efficient and individually rational.

Note 4.3.2.2. It is well known that allocations from the Walrasian equilibrium are both Pareto efficient and individually rational [Arrow and Debreu].

Although the Walrasian equilibrium has a number of useful properties, it runs into some difficulties when addressing the fair division of emission permits. The first and most important issue is that the Walrasian equilibrium may cause agents to be overcompensated, meaning they profit from the trading scheme.

Definition 4.3.2.3. An agent is overcompensated at an allocation $(x_i, t_i)_{i \in I}$ if

$$v_i(x_i) + t_i > v_i(\omega).$$

An agent is overcompensated if he receives a utility that makes he better off than if he were to consume the entire common resource on his own. Overcompensation contradicts the "fairness" of fair division because agents may use the trading system to their benefit. To address this overcompensation issue, Carsten Helm (2008) introduced a fourth equity criterion, the stand-alone upper bound, which will be discussed in the next section.

The second issue with the Walrasian equilibrium is that it does not guarentee an envy-free allocation. If allocations are not envy-free, agents may not be willing to participate in the trading scheme because they would rather argue for a larger portion of the emission permits (Remark 4.1.3.7). This will be discussed in Section 4.3.5.

4.3.3 The Fourth Equity Criterion

Carsten Helm [2008] introduces a fourth equity criterion to the fair division of emission permits, the stand-alone upper bound. The upper bound is used to ensure that there are no overcompensated agents within the trading scheme. The stand-alone upper bound is not a usual requirement for the Walrasian solution, so it is very important that we later prove our solution to the fair division problem meets this restriction.

Definition 4.3.3.1. An allocation $(x_i, t_i)_{i\in I}$ satisfies the stand-alone upper bound for E if

$$v_i(x_i) + t_i \leq v_i(w) \text{ for all } i \in I.$$

The stand-alone upper bound puts a limit on the total of the transfer payments that an agent can receive for selling his intitial entitlements. This guarentees that there are no overcompensated agents within the system. So no one is allowed to profit from exploiting the needs of other agents.

Example 4.3.3.2. Suppose Agent 1 and Agent 2 participate in an emission permit trading scheme with 100 permits that cost \$25 each. Both agents' utility functions are $U_i(x_i, t_i) = 10x_i + t_i$, where $t_i = p(\omega_i - x_i)$. The agent's beginning and final permit allocations and corresponding utilities are defined in the table below. Using Defini-

Table 4.3.1: Descriptive Table for Example 4.3.3.2

Agents	$v_i(\omega)$	ω_i	x_i	$v_i(x_i)$	$v_i(x_j)$	t_i	$U_i(x_i, t_i)$
1	1000	50	0	0	500	$1,250	1,250
2	1000	50	100	1000	500	$-1,250	-250

tion 4.3.3.1, we determine whether Agent 1 and Agent 2 satisfy the stand-alone upper bound.

Agent 1:

$U_1(x_1, t_1) \nleq U_1(\omega, 0)$ as $1,250 > 1,000$. Thus, Agent 1 violates the stand-alone upper bound.

Agent 2:

$U_2(x_2, t_2) \leq U_2(\omega, 0)$ as $-250 < 1,000$. Thus, Agent 2 satisfies the stand-alone upper bound.

The introduction of the stand-alone upper bound to a fair division trading scheme places a utility ceiling on each agent based on the amount of transfer payments they are allowed to receive. Once an agent reaches this utility ceiling he is considered *satiated*. An agent can be satiated either at the start with his initial entitlement and remain satiated through the trading, or he can start out unsatiated with his initial entitlement and become satiated through trading. It is important to note that many agents will never reach their satiation utility, but a trading scheme is still considered fair becuase the agents are individually rational i.e. at least as well off as at their initial entitlement.

Definition 4.3.3.3. An agent is satiated in his consumption of the common resource at his entitlement level if he has zero marginal utility for any $x_i > w_i$. As $w \geqslant w_i$, this implies

$$v_i(w_i) = v_i(w). \tag{4.3.3.4}$$

An agent who is satiated in his consumption of the common resource at his final allocation level has an utility function equal to his stand-alone upper bound,

$$U_i(x_i, t_i) = v_i(x_i) + t_i = v_i(w). \tag{4.3.3.5}$$

Proposition 4.3.3.6. *Let $(x_i, t_i)_{i\in I}$ be a solution to the fair division problem E that satisfies individual rationality and the stand-alone upper bound. Then all agents who are satiated in the usage of the common resource at their entitlement level receive an allocation (x_i, t_i), where $t_i = v_i(w) - v_i(x_i)$.*

Proof. Let $(x_i, t_i)_{i\in I}$ be a solution F to E that satisfies individual rationality and the stand-alone upper bound, and suppose that Agent i is satiated. By the definition of individual raitonality and the stand-alone upper bound, we know

$$v_i(\omega_i) \leqslant v_i(x_i) + t_i \leqslant v_i(w) \text{ for all } i \in I.$$

As $i \in I$ is satiated, the marginal utility for Agent i of any $x_i > \omega_i$ is zero. Thus $v_i(\omega_i) = v_i(w)$. Therefore,

$$t_i = v_i(w) - v_i(x_i) \text{ for all } i \in I.$$

□

Thus we know how to find t_i in terms of $v_i(w)$ and $v_i(x_i)$ for satiated agents. In the next section, I will show how to find t_i for all agents, those who are satiated and are not satiated with the bounded Walrasian solution.

4.3.4 The Bounded Walrasian Solution

To find a solution that satisfy the fourth equity criterion, the stand-alone upper bound, Helm (2008) constructed the bounded Walrasian solution, which is a variation of the Walrasian equilibrium.

The bounded Walrasian solution requires that all agents satisfy the stand-alone upper bound while still maximizing utility across all agents, i.e., guaranteeing Pareto efficiency. To maximize utility, yet not violate the upper bound agents may receive either:

(1) their stand-alone utility and the excess utility is redistributed or

(2) their utility-maximizing transfer payment plus some redistributed utility.

Which transfer payment each agent receives depends on whether the agent is ever satiated or not. Agents who are satiated with their intitial entitlement or receive their stand-alone utility through redistribution (those in the set S) receive option 1, i.e., $t_i = v_i(\omega - v_i(x_i)$. Agents who are never satiated (those in the set $I \backslash S$) receive option 2, i.e., $t_i = p(\omega_i - x_i) + \lambda$. The following definition lays out a minimum test that determines whether an allocation has appropriate transfer payments to be considered a bounded Walrasian solution.

Definition 4.3.4.1. Let the allocation $\mathbf{x} = (x_i)_{i\in I}$ and the price $p \in R_+$ constitute the Walrasian equilibrium. Consider some transfer payments $(t_i)_{i\in I}$ with the constraint that $\sum_{i\in I} t_i = 0$. Let $S = \{k \in I | t_k = v_k(w) - v_k(x_k)\}$. If for all $i \in I$,

$$t_i = \min\left[v_i(\omega) - v_i(x_i), p(\omega_i - x_i) + \frac{\sum_{k\in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]}{|I\backslash S|}\right],$$

then $(x_i, t_i)_{i\in I}$ is a bounded Walrasian solution.

We will refer to

$$t_i = \min\left[v_i(\omega) - v_i(x_i), p(\omega_i - x_i) + \frac{\sum_{k\in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]}{|I\backslash S|}\right]$$

from Definition 4.3.4.1 as the minimum test for an allocation, $(x_i, t_i)_{i\in I}$. If the t_i "passes" the minimum test, i.e. the minimum test is true for all $i \in I$, then the allocation is a bounded Walrasian solution.

Note 4.3.4.2. Notice that within Definition 4.3.4.1 the redistribution is defined as

$$\lambda = \frac{\sum_{k\in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]}{|I\backslash S|}.$$

We know the minimum test suffices for determining whether a fair division allocation is a bounded Walrasian solution or not for both satiated and unsatiated agents. If an agent is satiated the transfer payment that brings him up to his stand-alone utility is smaller than his utility maximizing transfer payment. If an agent is unsatiated his utility maximizing transfer payment plus some equally redistributed utility is smaller than the transfer payment that brings him up to his stand-alone utility.

Remark 4.3.4.3. It is important to note that how λ redistributes the excess utility is not an equity criterion. λ should be an equal per capita redistribution of utility, meaning the total amount of utility to be redistributed is divided by the number of unsatiated agents, and then each unsatiated agent receive an equal portion, instead of a proportional redistribution, where each agent receives a certain number of emission permits in proportion to their current emission levels. Proportional and therefore unequal redistribution could possibly lead to a violation of envy-freeness. Helm also notes that "the funds to be reallocated arise from a free service of members of S, from which the other agents should benefit equally" [Helm 2008].

Although Definition 4.3.4.1 gives us a condition for determining whether an allocation is the bounded Walrasian solution or not, to actually find the bounded Walrasian solution we must follow the following algorithm.

Algorithm 4.3.4.4. (The Bounded Walrasian Solution Algorithm).

Let the allocation $\mathbf{x} = (x_i)_{i \in I}$ and the price $p \in R_+$ constitute the Walrasian equilibrium. Consider transfer payments that maximize each agent's utility at the Walrasian equilibrium, i.e., $t_{i,1} = p(\omega_i - x_i)$ with the constraint that $\sum_{i \in I} t_{i,1} = 0$.

Step 1: If this free market allocation $(x_i, t_{i,1})_{i \in I}$, passes the minimum test (4.3.4.1), then we have a bounded Walrasian solution, and we are done.

If the allocation $(x_i, t_{i,1})_{i \in I}$ fails the minimum test, there is at least one agent who is overcompensated. We must redistribute the excess utility from overcompensated agents to unsatiated agents, i.e.,

$$\lambda_1 = \frac{\sum_{k \in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]}{|I \setminus S|},$$

where $\sum_{k \in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]$ is the excess utility of the overcompensated agents. Thus, agents who were overcompensated now receive a transfer payment that brings them up to their stand-alone utility, i.e., $t_{i,2} = v_i(\omega) - v_i(x_i)$, and agents who were unsatiated now receive their utility maximizing transfer payment plus the equal redistribution, i.e., $t_{i,2} = p(\omega_i - x_i) + \lambda_1$.

Step 2: If this new allocation $(x_i, t_{i,2})_{i \in I}$, passes the minimum test (4.3.4.1), then we have a bounded Walrasian solution, and we are done.

If the allocation $(x_i, t_{i,2})_{i \in I}$ fails the minimum test, there is at least one agent who is overcompensated. We must redistribute the excess utility from overcompensated agents to unsatiated agents, i.e.,

$$\lambda_2 = \frac{\sum_{k\in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]}{|I\setminus S|},$$

where $\sum_{k\in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]$ is the excess utility of the overcompensated agents. Thus, agents who were overcompensated now receive a transfer payment that brings them up to their stand-alone utility, i.e., $t_{i,3} = v_i(\omega) - v_i(x_i)$, and agents who were unsatiated now receive their utility maximizing transfer payment plus the equal redistribution, i.e., $t_{i,3} = p(\omega_i - x_i) + \lambda_2$.

$$\vdots$$

Step $n-1$: If this new allocation $(x_i, t_{i,n-1})_{i\in I}$ passes the minimum test (4.3.4.1), then we have a bounded Walrasian solution, and we are done.

If the allocation $(x_i, t_{i,n-1})_{i\in I}$ fails the minimum test, there is at least one agent who is overcompensated. We must redistribute the excess utility from overcompensated agents to unsatiated agents, i.e.,

$$\lambda_{n-1} = \frac{\sum_{k\in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]}{|I\setminus S|},$$

where $\sum_{k\in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]$ is the excess utility of the overcompensated agents. Thus, agents who were overcompensated now receive a transfer payment that brings them up to their stand-alone utility, i.e., $t_{i,n} = v_i(\omega) - v_i(x_i)$, and agents who were unsatiated now receive their utility maximizing transfer payment plus the equal redistribution, i.e., $t_{i,n} = p(\omega_i - x_i) + \lambda_{n-1}$.

Step n: This new allocation $(x_i, t_{i,n})_{i\in I}$ must pass the minimum test (4.3.4.1), i.e. we have a bounded Walrasian solution, and we are done.

We will later prove that Algorithm 4.3.4.4 must terminate with a bounded Walrasian solution, and therefore the bounded Walrasian solution exists. But first to better understand how the Algorithm works, we provide an example of a trading scheme between 4 agents. The example follows the step-by-step process of Algorithm 4.3.4.4 to eventually find the bounded Walrasian solution.

Example 4.3.4.5. Suppose 4 agents participate in a trading scheme. The price, $p = 0.012$, is defined as the amount where all 4 agents' marginal utilities are equal. Let $\omega = 200$. Refer to the Appendix for the utility fuctions of each agent.

Step 1: Find the free market allocation $(x_i, t_{i,1})_{i\in I}$, where $t_{i,1} = p(\omega_i - x_i)$. We maximize utility across all agents bound by the constraint that $\sum_{i\in I} t_{i,1} = 0$. Please refer to Table 7.2.1 in the Appendix.

Since Agent 2 does not pass the minimum test, i.e., $p(\omega_2 - x_2) > v_2(\omega) - v_2(x_2)$, we know that Agent 2 is overcompensated. Therefore the excess utility, 0.07, must be redistributed between the three agents who are unsatiated, i.e., $p(\omega_i - x_i) < v_i(\omega) - v_i(x_i)$. So $\lambda_1 = 0.07/3 = 0.022$, and λ_1 is added to $t_{i,1}$ to Agents 1, 3, and 4, i.e., $t_{i,2} = p(\omega_i - x_i) + \lambda_1$.

Step 2: Determine whether the new allocation $(x_i, t_{i,2})_{i\in I}$ satisfies Definition 4.3.4.1. Please refer to Table 7.2.2 in the Appendix.

Since Agent 4 does not pass the minimum test, i.e., $p(\omega_4 - x_4) + \lambda_1 > v_4(\omega) - v_4(x_4)$, we know that Agent 4 is overcompensated. Therefore the excess utility, 0.02, must be redistributed between the two agents who are unsatiated, i.e., $p(\omega_i - x_i) + \lambda_1 <$

$v_i(\omega) - v_i(x_i)$. So $\lambda_2 = 0.02/2 = 0.01$, and λ_2 is added to $t_{i,2}$ to Agents 1, and 3 i.e., $t_{i,3} = p(\omega_i - x_i) + \lambda_2$.

Step 3: Determine whether the new allocation $(x_i, t_{i,3})_{i\in I}$ satisfies Definition 4.3.4.1. Please refer to Table 7.2.3 in the Appendix.

Since all agents pass the minimum test, the allocation $(x_i, t_{i,3})_{i\in I}$ is the bounded Walrasian solution.

Now that we have shown an example of how to find the bounded Walrasian solution, we must prove that Algorithm 4.3.4.4 always terminates and that the termination is a bounded Walrasian solution, and thus the bounded Walrasian solution exists.

Proposition 4.3.4.6. *Given some fair division problem $E \equiv \{U_i, \omega_i\}_{i\in I}$, the Algorithm 4.3.4.4 always terminates at a bounded Walrasian solution.*

Proof. Suppose $E \equiv \{U_i, \omega_i\}_{i\in I}$ is a fair division problem for a scarce common resource, ω, among n agents. Since ω is scarce, we know that not all agents can be satiated at once, i.e. for any allocation of ω, there exists some agent $i \in I$ who is not satiated. Suppose we have $(x_i)_{i\in I}$ and p of the Walrasian equilibrium and an initial $t_{i,1} := p(\omega_i - x_i)$. Let

$$S = \{r \in S | t_r \geq v_r(\omega) - v_r(x_r)\}$$

$$I \backslash S = \{s \in I \backslash S | t_s \leq v_s(\omega) - v_s(x_s)\}$$

Follow the process of Algorithm 4.3.4.4, as follows.

Step 1: If $t_{i,1} \leq v_i(\omega) - v_i(x_i)$ for all agents $i \in I$, then $(x_i, t_{i,1})_{i\in I}$ is a bounded Walrasian solution, and we are done.

If not, let $t_{r,2} = v_r(\omega) - v_r(x_r)$ for all agents $r \in S$ such that $t_{r,1} \geq v_r(\omega) - v_r(x_r)$. Also, let $t_{s,2} = p(\omega_s - x_s) + \lambda_1$ for all agents $s \in I \backslash S$ such that $t_{s,1} \leq v_s(\omega) - v_s(x_s)$, where

$$\lambda_1 = \frac{\sum_{r \in S}[p(\omega_r - x_r) - (v_r(\omega) - v_r(x_r))]}{|I \backslash S|}.$$

Step 2: If $t_{i,2} \leq v_i(\omega) - v_i(x_i)$ for all agents $i \in I$, then $(x_i, t_{i,2})_{i \in I}$ is a bounded Walrasian solution, and we are done.

If not, let $t_{r,3} = v_r(\omega) - v_r(x_r)$ for all agents $r \in S$ such that $t_{r,2} \geq v_r(\omega) - v_r(x_r)$. Also, let $t_{s,3} = p(\omega_s - x_s) + \lambda_2$ for all agents $s \in I \backslash S$ such that $t_{s,2} \leq v_s(\omega) - v_s(x_s)$, where

$$\lambda_2 = \frac{\sum_{r \in S}[p(\omega_r - x_r) - (v_r(\omega) - v_r(x_r))]}{|I \backslash S|}.$$

$$\vdots$$

Step $n-1$: If $t_{i,n-1} \leq v_i(\omega) - v_i(x_i)$ for all agents $i \in I$, then $(x_i, t_{i,n-1})_{i \in I}$ is a bounded Walrasian solution, and we are done.

If not, let $t_{r,n} = v_r(\omega) - v_r(x_r)$ for all agents $r \in S$ such that $t_{r,n-1} \geq v_r(\omega) - v_r(x_r)$. Also, let $t_{s,n} = p(\omega_s - x_s) + \lambda_{n-1}$ for all agents $s \in I \backslash S$ such that $t_{s,n-1} \leq v_s(\omega) - v_s(x_s)$, where

$$\lambda_{n-1} = \frac{\sum_{r \in S}[p(\omega_r - x_r) - (v_r(\omega) - v_r(x_r))]}{|I \backslash S|}.$$

Step n: Since there are n agents participating in the fair division, if we have not yet found a bounded Walrasian solution we know all other agents, besides Agent n, are satiated with their stand-alone utility as they all receive a transfer payment $t_{i,n} = v_i(\omega) - v_i(x_i)$. But since all agents are satiated, except for one, we know the one agent must be unsatiated by the assumption that the common resource, ω, is scarce, i.e. there exists some agent $i \in I$ who is not satiated. This remaining unsatiated agent must be Agent n. Therefore, we know Agent n's transfer payment is

$$t_{n,n} = p(\omega_n - x_n) + \lambda_{n-1}$$
$$\leq v_n(\omega) - v_n(x_n)$$

since Agent n is in $I \backslash S$. Thus, $t_{i,n} \leq v_i(\omega) - v_i(x_i)$ for all $i \in I$. Thus, $(x_i, t_{i,n})_{i \in I}$ is a bounded Walrasian solution.

Thus, Algorithm 4.3.4.4 always terminates at a bounded Walrasian solution. Also, the process is guarenteed to terminate since the common resource is scarce. □

Although the bounded Walrasian solution satisfies the stand-alone upper bound since the highest utility an agent may receive is their stand-alone utility, it still does not address the issue of envy-freeness discussed in the last section. Again, refer to Remark 4.1.3.7. In the next section, we examine why the Walrasian equilibrium (and therefore the bounded Walrasian solution) does not guarentee envy-freeness. Helm proposes a variation to envy-freeness, envy-freeness with respect to a compensation rule, which shows that agents do not want one another's final permit allocation, x_i.

4.3.5 Envy-freeness with Respect to a Compensation Rule

We now return to the discussion of the third equity criterion, envy-freeness (Definition 4.3.1.4), from Section 4.1.3. Consider how envy-freeness applies to Helm's model.

According to Definition 4.3.1.4, within Helm's model an allocation $(x_i, t_i)_{i \in I}$ is envy-free if for all pairs of agents $i, j \in I$,

$$v_i(x_i) + t_i \geq v_i(x_j) + t_j. \tag{4.3.5.1}$$

Suppose the agents' transfer payments are based on the Walrasian equilibrium, i.e., their revenue or expenditure for the emission permits. Within the Walrasian equilibrium, revenue and expenditure are both calculated by multiplying the change in an agent's permit quantity by the market price, $t_i = p(\omega_i - x_i)$. So substituting this into Equation (4.3.5.1), we get

$$v_i(x_i) + p(\omega_i - x_i) \geq v_i(x_j) + p(\omega_j - x_j) \quad (4.3.5.2)$$

for all pairs of agents $i, j \in I$.

Remark 4.3.5.3. Notice that $t_i = p(\omega_i - x_i)$, is a function of ω_i and x_i (p is a constant).

The following example illustrates a violation of envy-freeness. We will use this example to better understand what conditions lead to envy-freeness or the violation of envy-freeness.

Example 4.3.5.4. Suppose Agent 1 and Agent 2 participate in an emission permit trading scheme with 100 permits that each cost \$25. Suppose Agents 1 and 2 have the same preferences for emission permits, i.e., $v_1(x_i) = v_2(x_i)$ for all x_i. The agents' beginning and final permit allocations and utilities are defined in the table below.

Table 4.3.2: Descriptive Table for Example 4.3.5.4

Agents	ω_i	x_i	$v_i(x_i)$	$v_i(x_j)$	t_i
1	60	50	500	500	\$250
2	40	50	500	500	\$-250

Using the usual definition of envy-freeness (4.3.1.4) with reference to Equation (4.3.5.2), we determine whether Agent 1 and Agent 2 are envy-free.

Agent 1:

$$v_1(x_1) + p(\omega_1 - x_1) = 500 + 25(10) = 750$$

$$v_1(x_2) + p(\omega_2 - x_2) = 500 + 25(-10) = 250.$$

$750 \geq 250$. So Agent 1 is envy-free.

Agent 2:

$$v_2(x_2) + p(\omega_2 - x_2) = 500 + 25(-10) = 250$$

$$v_2(x_1) + p(\omega_1 - x_1) = 500 + 25(10) = 750.$$

$250 \ngeq 750$. So Agent 2 envies Agent 1.

Thus, Example 4.3.5.4 violates the usual definition of envy-freeness within Helm's emission permit trading scheme. But why does this violation occur?

It is not unusual that an Walrasian equilibrium violates the usual definition of envy-freeness. Most markets are not envy-free because agent have different budget sets–a set of possible allocations that the agent can afford. In Example 4.3.5.4, the agents' budget sets are ω_i. The agents have the same utility functions and end up with final allocations of the same number of emission permits. The only difference between the two agents is that Agent 1 has $\omega_1 = 60$ that is larger than Agent 2's $\omega_2 = 40$. Because of this difference in their ω_i's, Agent 2 envies Agent 1. In order for both agents to be envy-free, they must have equal initial entitlements, i.e. $\omega_1 = \omega_2 = 50$.

Now, consider another example where the agents have equal initial entitlements, but different utility function. The following example illustrates conditions that will most likely lead to envy-freeness.

Example 4.3.5.5. Suppose Agent 1 and Agent 2 participate in an emission permit trading scheme with 100 permits that each cost \$25. Suppose Agent 1 has a higher marginal utility than Agent 2 up to 55 permits. The agents' beginning and final permit allocations and utilities are defined in the table below.

Table 4.3.3: Descriptive Table for Example 4.3.5.5

Agents	ω_i	x_i	$v_i(x_i)$	$v_i(x_j)$	t_i
1	50	55	600	490	$-125
2	50	45	450	550	$125

Using the usual definition of envy-freeness (4.3.1.4) we determine whether Agent 1 and Agent 2 are envy-free.

Agent 1:

$$v_1(x_1) + p(\omega_1 - x_1) = 600 + 25(-5) = 725$$

$$v_1(x_2) + p(\omega_2 - x_2) = 490 + 25(5) = 615.$$

$725 \geq 615$. So Agent 1 is envy-free.

Agent 2:

$$v_2(x_2) + p(\omega_2 - x_2) = 450 + 25(5) = 575$$

$$v_2(x_1) + p(\omega_1 - x_1) = 550 + 25(-5) = 425.$$

$575 \geq 425$. So Agent 2 is envy-free.

Thus, both agents are envy-free, even though they have different utility functions. Although the agents may envy one another if the difference in their utility functions and final permit allocations is large enough, the main point is equal initial entitlements is a condition of envy-freeness for Helm's model.

We know that equal initial entitlements across agents is not a *politically* realistic solution for Helm's model because large and powerful countries, like the United States, would never agree to participate in the emission trading scheme. It is important that the participating countries believe the starting point (initial entitlements) and the ending point (final allocation) are both fair. Helm argues the bounded Walrasian solution guarentees the latter because it is Pareto efficient, individually rational, envy-free with respect to a compensation rule (4.3.5.6), and satisfies the stand-alone upper bound, which will be proved in Section 4.3.6. To guarentee the former, we must define a "fair" manner if dividing up the initial entitlements.

Even though agents may still envy one another's intitial entitlements, they are subject to a rule that distributes the initial permits, which can be justified as reasonable because the agents are "entitled" to different amounts of permits. Helm (2008) uses an interesting analogy to illustrate how entitlements can be unequal and still be "fair."

> Suppose that the wife of a deceased husband receives a share of the inheritance that is unambiguously more valuable than the share of a remote relative. Then the [usual definition] of envy-freeness is not applicable because the two are unequally entitled to the inheritance. [Helm 2008]

There is an argument that the United States is entitled to a larger potion of the initial emission permits than Switzerland because the U.S. has the largest economy in the world. We will not delve into this volatile debate, but simply draw on the method of the most widely accepted international climate change agreement, the Kyoto Protocol. The Kyoto Protocol required countries to reduce their emission to 5.2% less than their 1990 emission amount. Thus, since we have decided on a "fair" rule for reduction, agents can justify their initial entitlement to one another.

We have shown that the usual definition of envy-freeness is not useful to Helm's model by the above justification because envy-freeness only works if the agents have equal initial entitlements. Still, Helm wants to impose a variation of envy-freeness to show that agents do not want one another's final permit allocation, x_i. Helm's variation to envy-freeness is called "envy-freeness with respect to a compensation rule."

Definition 4.3.5.6. Let $\mathbf{x} \equiv (x_1, ..., x_i, ..., x_j, ..., x_n)$ be an allocation vector of the common resource, and denote by $\mathbf{\hat{x}_{ij}} \equiv (x_1, ..., x_j, ..., x_i, ..., x_n)$ the same vector with transposed entries for agents i and j. A compensation rule is a function $T_i(\mathbf{x})$ that associates for every agent $i \in I$ to every $\mathbf{x}$ a level of compensation to be paid or received. It is required that $\sum_{i \in I} T_i(\mathbf{x}) = 0$. An allocation $(x_i, T_i(\mathbf{x}))_{i \in I}$ from justifiable entitlements to the common resource is envy-free with respect to the compensation rule $T_i(\mathbf{x})$ for the fair division problem E if for all pairs of agents, $i, j \in I$:

$$v_i(x_i) + T_i(\mathbf{x}) \geqslant v_i(x_j) + T_i(\mathbf{\hat{x}_{ij}}).$$

Note: This definition is Helm's third equity criterion for his fair division model, instead of envy-freeness.

To better understand what this definition is saying we consider the situation where $t_i = p(\omega_i - x_i)$ as in Equation (4.3.5.2). But this time, t_i is replaced with our compensation rule, $T_i(\mathbf{x})$.

Remark 4.3.5.7. Refering back to Remark 4.3.5.3, this time $T_i(\mathbf{x}) = p(\omega_i - x_i)$ is only a function of x_i (p and ω_i are constants). When we consider $\mathbf{\hat{x}_{ij}}$, $T_i(\mathbf{\hat{x}_{ij}}) = p(\omega_i - x_j)$, so $T_i(\mathbf{\hat{x}_{ij}})$ is a function of x_j.

We now redefine the usual definition of envy-freeness within Helm's model to envy-freeness within Helm's model with respect to the $T_i(\mathbf{x}) = p(\omega_i - x_i)$ compensation rule. To do this we substitute $T_i(\mathbf{x}) = p(\omega_i - x_i)$ for t_i and $T_i(\hat{\mathbf{x}}_{\mathbf{ij}}) = p(\omega_i - x_j)$ for t_j into Equation (4.3.5.1), and get

$$v_i(x_i) + p(\omega_i - x_i) \geqslant v_i(x_j) + p(\omega_i - x_j) \tag{4.3.5.8}$$

for all pairs of agents $i, j \in I$.

We return again to Example 4.3.5.4 to better illustrate the what envy-freeness with respect to a compensation rule means.

Example 4.3.5.9. Refer to the table in Example 4.3.5.4. Using the definition of envy-freeness with respect to a compensation rule (4.3.5.6) with reference to Equation 4.3.5.8, we determine whether Agent 1 and Agent 2 are envy-free with respect to a compensation rule.

Agent 1:

$$v_1(x_1) + p(\omega_1 - x_1) = 500 + 25(10) = 750$$

$$v_1(x_2) + p(\omega_1 - x_2) = 500 + 25(10) = 750.$$

$750 \geq 750$. So Agent 1 is envy-free with respect to a compensation rule.

Agent 2:

$$v_2(x_2) + p(\omega_2 - x_2) = 500 + 25(-10) = 250$$

$$v_2(x_1) + p(\omega_2 - x_1) = 500 + 25(-10) = 250.$$

$250 \geq 250$. So Agent 2 is envy-free with respect to a compensation rule.

To further illustrate the difference between the usual definition of envy-freeness and envy-freeness with respect to a compensation rule, we consider another example of an emission trading scheme.

Example 4.3.5.10. Suppose China and Mongolia are in a global emission permits trading scheme with the 192 members of the United Nations. Mongolia will have a very low initial entitlement of emission permits, say 2, compared to China's 20, by some redetermined justifable initial entitlement rule. Let $p = \$10$.

Table 4.3.4: Descriptive Table for Example 4.3.5.10

Agents	ω_i	x_i	t_i	$T_i(\mathbf{x})$	$T_i(\mathbf{\hat{x}_{ij}})$
Mongolia	2	1	10	10	-160
China	20	18	20	20	190

Does Mongolia envy China using the usual definition of envy-freeness (4.3.5.1)?:

$$U_M(1,10) \ngeq U_M(18,20)$$

Mongolia envies China as the utility functions are monotonic by construction.

Does Mongolia envy China using envy-freeness with respect to a compensation rule, where the compensation rule is $T_i(\mathbf{x}) = p(\omega_i - x_i)$ (4.3.5.8)?:

We calculate $T_M(\mathbf{\hat{x}_{CM}}) = p(\omega_M - x_C) = 10(2-18) = -160$. So,

$$U_M(1,10) \geq^? U_M(18,-160).$$

Unlike the usual definition, it is unclear whether the above inequality is true or not without more information. But this is the sort of outcome we are looking for with envy-freeness with respect to a compensation rule. The uncertainty stems from the question of whether Mongolia will be willing to pay the very large total cost of these permits, because it would need to buy 18 additional permits to match China's final allocation number, $T_M(\hat{\mathbf{x}}_{\mathbf{CM}})$.

Quite often this large additional cost is less than the additional utility gained from having these permits, which means Mongolia would be envy-free with respect to a compensation rule. But if the cost is worth it, Mongolia would have simply choosen $x_M = 18$ as its final allocation because each agent maximizes its own utility.

Now that we understand what envy-freeness with respect to a compensation rule, we consider envy-freeness with respect to the BWS compensation rule.

Definition 4.3.5.11. An allocation $(x_i, T_i(\mathbf{x}))_{i \in I}$, where

$$T_i(\mathbf{x}) = \min\left[v_i(\omega) - v_i(x_i), p(\omega_i - x_i) + \frac{\sum_{k \in S}[p(\omega_k - x_k) - (v_k(\omega) - v_k(x_k))]}{|I \setminus S|}\right],$$

is envy-free with respect to the BWS compensation rule for justifiable entitlements if for all pairs of agents, $i, j \in I$:

$$v_i(x_i) + T_i(\mathbf{x}) \geqslant v_i(x_j) + T_i(\hat{\mathbf{x}}_{\mathbf{ij}}).$$

4.3.6 Does the Bounded Walrasian Solution Satisfy the Four Equity Criteria?

Now, we must show that the bounded Walrasian solution satisfies all four equity criteria, Pareto efficiency, individual rationality, the stand-alone upper bound, and envy-freeness with respect to a compensation rule.

Theorem 4.3.6.1. *The bounded Walrasian solution satisfies:*

1) Pareto efficiency,

2) individual rationality,

3) the stand-alone upper bound, and

4) envy-freeness with respect to the BWS compensation rule for justifiable entitlements.

Proof. Suppose the allocation $(x_i, t_i)_{i \in I}$ is a bounded Walrasian solution. By the definition of the BWS (4.3.4.1), we know the allocation $(x_i)_{i \in I}$ and the price $p \in R_+$ are a Walrasian equilibrium. Let

$$S = \{r \in S | t_r \geq v_r(\omega) - v_r(x_r)\}$$

$$I \backslash S = \{s \in I \backslash S | t_s \leq v_s(\omega) - v_s(x_s)\}$$

where S is the set of agents who are satiated, and $I \backslash S$ is the set of agents who are unsatiated.

1) **Pareto efficiency.** We must show the BWS allocation $(x_i, t_i)_{i \in I}$ is Pareto efficient. By Note 4.3.2.2, we know the allocation $(x_i)_{i \in I}$ is Pareto efficient. Further, by Proposition 4.3.1.6 we know $(x_i)_{i \in I}$ maximizes utility across all agents. Since the BWS simply redistributes utility among the agents, but does not change the total utility we know that $(x_i, t_i)_{i \in I}$ also maximizes the sum of total utility for all agents. So again by Proposition 4.3.1.6, the BWS allocation $(x_i, t_i)_{i \in I}$ is Pareto efficient.

2) **Individual rationality.** We must show the BWS allocation $(x_i, t_i)_{i \in I}$ is individually rational, i.e.

$$v_i(x_i) + t_i \geq v_i(\omega_i) \text{ for all } i \in I.$$

To prove this we will prove individual rationality with two cases, for those who are satiated and for those who are not.

Case 1: Suppose $j \in S$. Under the bounded Walrasian solution, we know all agents in

S have a transfer payment $t_j = v_j(w) - v_j(x_j)$ by Algorithm 4.3.4.4. Then

$$v_j(x_j) + t_j = v_j(x_j) + v_j(w) - v_j(x_j)$$
$$= v_j(w) \text{ for all } j \in S.$$

Case 2: Suppose $i \in I \backslash S$. Under the bounded Walrasian solution, we know all agents in $I \backslash S$ have a transfer payment $t_i = p(\omega_i - x_i) + \lambda$, where $\lambda \geq 0$ by Algorithm 4.3.4.4. Then

$$v_i(x_i) + t_i = v_i(x_i) + p(\omega_i - x_i) + \lambda$$
$$\geq v_i(x_i) + p(\omega_i - x_i) \text{ as } \lambda \geq 0$$
$$\geq v_i(\omega_i) \text{ for all } i \in I \backslash S,$$

since we know the allocation $(x_i)_{i \in I}$ is individually rational by Note 4.3.2.2.

Thus, we have shown $v_i(x_i) + T_i(x) \geq v_i(\omega_i)$ for all $i \in I$.

3) **Stand-alone upper bound.**We must show the BWS allocation $(x_i, t_i)_{i \in I}$ satisfies the stand-alone upper bound, i.e.,

$$v_i(x_i) + t_i \leq v_i(w) \text{ for all } i \in I.$$

To prove this we will prove the BWS allocation satisfies the stand-alone upper bound in two cases, for those who are satiated and for those who are not.

Case 1: Suppose $j \in S$. Under the bounded Walrasian solution, we know all agents in

S have a transfer payment $t_j = v_j(w) - v_j(x_j)$ by Algorithm 4.3.4.4. Then

$$\begin{aligned} v_j(x_j) + t_j &= v_j(x_j) + v_j(w) - v_j(x_j) \\ &= v_j(w) \\ &\leq v_j(w) \text{ for all } j \in S. \end{aligned}$$

Case 2: Suppose $i \in I \backslash S$. Since i is unsatiated, we know $U_i(x_i, t_i) < v_i(w)$ for all $i \in I \backslash S$.

Therefore, we have shown that $v_i(x_i) + t_i \leq v_i(w)$ for all $i \in I$.

4) **Envy-freeness with respect to the BWS compensation rule for justifiable entitlements.** We must show the BWS allocation $(x_i, T_i(\mathbf{x}))_{i \in I}$ is envy-free with respect to the BWS compensation rule for justifiable entitlements, i.e.,

$$v_i(x_i) + T_i(\mathbf{x}) \geqslant v_i(x_j) + T_i(\mathbf{\hat{x}_{ij}}).$$

To prove this we will prove the BWS allocation is envy-free with respect to the BWS compensation rule in two cases, for those who are satiated and for those who are not.

Case 1: Suppose $j \in S$. Under the bounded Walrasian solution, we know all agents in S have a transfer payment $T_j(\mathbf{x}) = v_j(w) - v_j(x_j)$ by Algorithm 4.3.4.4. Suppose by way of contradiction that Agent j envies Agent i with respect to the BWS compensation rule, where $i \in I$. Then

$$v_j(x_j) + T_j(\mathbf{x}) < v_j(x_i) + T_j(\mathbf{\hat{x}_{j,i}})$$

By the definition of the stand-alone upper bound, we know $T_j(\mathbf{\hat{x}_{j,i}}) \leq v_j(\omega) - v_j(x_i)$.

By substitution for $T_j(\mathbf{x})$ and $T_j(\hat{\mathbf{x}}_{\mathbf{j},\mathbf{i}})$, we find

$$v_j(x_j) + v_j(w) - v_j(x_j) < v_j(x_i) + v_j(\omega) - v_j(x_i),$$

i.e.

$$v_j(\omega) < v_j(\omega).$$

This is a contradiction. Thus, Agent j does not envy Agent i with respect to the BWS compensation rule, where $i \in I$.

Case 2: Suppose $i \in I \backslash S$. We know $\mathbf{x} = (x_i)_{i \in I}$ and p constitute a Walrasian equilibrium by the definition of bounded Walrasian solution, i.e.

$$v_i(x_i) + p(\omega_i - x_i) \geq v_i(x_j) + p(\omega_i - x_j) \quad \text{for all} \quad j \in I. \tag{4.3.6.2}$$

Since Agent j's final allocation, x_j, is also part of the Walrasian equilibrium, we know

$$v_j(x_j) + p(\omega_j - x_j) \geq v_j(x_i) + p(\omega_j - x_i). \tag{4.3.6.3}$$

We must show $v_i(x_i) + T_i(\mathbf{x}) \geq v_i(x_j) + T_i(\hat{\mathbf{x}}_{\mathbf{i},\mathbf{j}})$ for all $j \in I$. Let

$$\lambda = \frac{\sum_{k \in S} \left[p(w_k - x_k) - (v_k(w) - v_k(x_k)) \right]}{|I \backslash S|}$$

denote the final amount of utility that is redistributed from agents in S to agents in $I \backslash S$ at the last step in Algorithm 4.3.4.4 and $\hat{\lambda}_{i,j}$ denote the amount of utility that is redistributed to i if x_i and x_j are transposed.

Remark 4.3.6.4. It is important to note that although λ and $\hat{\lambda}_{i,j}$ do not have any x_i's and x_j's to transpose, but there is a concern that when they are transposed Agent j receiving x_i may force Agent j to become satiated. If this transpose causes Agent j

to become satiated, this agent will move from $I \backslash S$ into S, and therefore increase the quantity $\sum_{k \in S} \left[p(w_k - x_k) - (v_k(w) - v_k(x_k)) \right]$ and decrease $|I \backslash S|$, i.e., if the transpose causes Agent j to enter S, and $\hat{\lambda}_{i,j} > \lambda$. We will show that this never occurs.

To prove $i \in I \backslash S$ is envy-free with respect to the BWS compensation rule with justifiable entitlement, we must show for all $j \in I$

$$v_i(x_i) + p(\omega_i - x_i) + \lambda \geq v_i(x_j) + p(\omega_i - x_j) + \hat{\lambda}_{i,j}.$$

We must know whether j is in S or $I \backslash S$, so we break the proof into two parts: (a) if $j \in I \backslash S$ and (b) if $j \in S$.

Part (a): Suppose $j \in I \backslash S$. We know

$$\begin{aligned} 0 &\geq p(\omega_j - x_j) + \lambda - [v_j(\omega) - v_j(x_j)] \\ &\geq p(\omega_j - x_j) - [v_j(\omega) - v_j(x_j)] \quad \text{since} \quad \lambda \geq 0 \\ &\geq p(\omega_j - x_i) - [v_j(\omega) - v_j(x_i)] \end{aligned}$$

by subtracting $v_j(\omega)$ from both sides of Equation (4.3.6.3). Thus, j remains in the set $I \backslash S$ because the right hand of the inequality is even smaller than all of the left. So we know $\min \left[v_i(\omega) - v_i(x_j), p(\omega_i - x_j) + \hat{\lambda}_{i,j} \right] = p(\omega_i - x_j) + \hat{\lambda}_{i,j}$. Now, we must show

$$v_i(x_i) + p(\omega_i - x_i) + \lambda \geq v_i(x_j) + p(\omega_i - x_j) + \hat{\lambda}_{i,j}.$$

But by Equation (4.3.6.2), it would be sufficient to prove $\lambda \geq \hat{\lambda}_{i,j}$. Since j remains in $I \backslash S$, there is no change to the agents in S, so there also no change to the amount of redistribution, i.e. $\lambda = \hat{\lambda}_{i,j}$.

Part (b): Suppose $j \in S$. We must show

$$v_i(x_i) + p(\omega_i - x_i) + \lambda \geq v_i(x_j) + \min\left[v_i(\omega) - v_i(x_j), p(\omega_i - x_j) + \hat{\lambda}_{i,j}\right].$$

If Agent j receives x_i instead of x_j, we know $\sum_{k\in S}\left[p(w_k - x_k) - (v_k(w) - v_k(x_k))\right]$ will decrease by Equation (4.3.6.3). Thus $\hat{\lambda}_{i,j} \leq \lambda$.

Since $i \in I\backslash S$, we know

$$\begin{aligned} v_i(\omega) &\geq v_i(x_i) + p(\omega_i - x_i) + \lambda \\ &\geq v_i(x_i) + p(\omega_i - x_i) + \hat{\lambda}_{i,j} \quad \text{since} \quad \lambda \geq \hat{\lambda}_{i,j} \\ &\geq v_i(x_j) + p(\omega_i - x_j) + \hat{\lambda}_{i,j} \quad \text{by Equation} \quad (4.3.6.3). \end{aligned}$$

Thus, $v_i(\omega) - v_i(x_j) \geq p(\omega_i - x_j) + \hat{\lambda}_{i,j}$. So, $\min\left[v_i(\omega) - v_i(x_j), p(\omega_i - x_j) + \hat{\lambda}_{i,j}\right] = p(\omega_i - x_j) + \hat{\lambda}_{i,j}$. Therefore,

$$v_i(x_j) + \min\left[v_i(\omega) - v_i(x_j), p(\omega_i - x_j) + \hat{\lambda}_{i,j}\right] = v_i(x_j) + p(\omega_i - x_j) + \hat{\lambda}_{i,j}.$$

Thus, as we know Equation (4.3.6.2) and $\lambda \geq \hat{\lambda}_{i,j}$,

$$v_i(x_i) + p(\omega_i - x_i) + \lambda \geq v_i(x_j) + p(\omega_i - x_j) + \hat{\lambda}_{i,j}.$$

□

Chapter 5

Data

This chapter examines the data used in our market simulation. In order to simulate the pollution levels and transfer payments the bounded Walrasian solution would produce, we must have consistent and fluid data. Therefore, we used only one source for each type of data that we gathered in hopes that although the data may not be perfectly accurate at least it will be proportional across countries, so we can compare the differences.

5.1 Marginal Abatement Cost Curves

The best tools for forecasting pollution reduction costs are marginal abatement cost curves (MACCs). MACCs are projected estimates of the annuals costs of both avoided and abated carbon emissions. This means they capture not only how much it will cost to do additional carbon abatement, but also how much emissions are "avoided" through intrinsic growth and technological advancement. The point of the MACC is to measure the cost of additional abatement beyond "business as usual." The "business as usual" projection estimates the growth in carbon emission, mostly from the growing demand for energy and transportation, as well as the deforestation of the tropics.

Consider Figure 7.2.1 in the Appendix, which is a MACC for Russia with a projected year of 2030. The vertical axis is the abatement cost per ton of carbon, and the horizontal axis is the potential abatement of carbon per year. Each vertical bar is a possible option the country has for reducing pollution, like recycling new waste or improving the installation in residential buildings. Each possible option has a corresponding potential abatement amount and cost. Therefore, the area of the bar can be thought of as the expenditure (or revenue) for abatement. Any potential abatement that occurs at a cost lower than €0 is considered a "negative" cost. Negative costs indicate a "net benefit or savings to the economy over the lifecycle of the option." [McKinsey & Company]. These bars that are below the x-axis correspond to green innovations that lead to net savings i.e., provide positive economic returns over time.

The MACCs we use are developed by McKinsey & Company. Although other think tanks have developed their own versions of abatement curves–EnerData, Bloomberg New Energy Finance–McKinsey & Company has made some of their curves publicly available. MACCs are extremely valuable not only to those participating in carbon trading systems, but energy companies (for deciding about long-term capital investment strategies) and policy-makers (for determining how much abatement a particular economy can afford and what technologies should be targeted for abatement). McKinsey & Company has made public MACCs for ten countries–Australia, Brazil, China, India, Poland, Russia, Sweden, Switzerland, United Kingdom, and United States–as well as a global MACC. These are the ten countries that we use in our market simulation. The global MACC is used to find the global market price for emission permits for one ton of CO_2. The MACCs are used to find how much each country can abate at a particular global market price.

McKinsey & Company's first step to developing their 2030 MACCs was finding their "business-as-usual" projection. How McKinsey developed each business-as-usual case varies from country to country depending on the information available. For the global MACC, they used a growth model and made very important assumption about future growth. If these assumptions were to change, the projection could be greatly affected. The first assumption is that annual GDP growths for developed and developing countries are 2.1% and 5.5%, respectively. Another is that global population growth is 0.9%, while 0.2% and 1.1% in developed and developing countries, respectively. The last assumption is that oil is $60 per barrel. McKinsey & Company chose all of these assumptions based on the International Energy Agency's (IEA) World Energy Outlook for 2007. They found that greenhouse gas emissions would increase by about 55% from 2005 to 2030. From this 2030 business-as-usual forecast, McKinsey evaluated the potential costs and benefits of more than 200 greenhouse gas abatement opportunities across 21 world regions focusing on 10 sectors. These sectors are power, petroleum and gas, cement, iron and steel, chemicals, transport, buildings, waste, forestry, and agriculture. They incorporated detailed information about abatement potential, investment and financing needs, and implementation scenarios to gain a dynamic understanding of how abatement reductions could occur.

The other ten MACCs for the countries were developed in a generally similar fashion, with differences based on the availability of information. Again, McKinsey & Company first developed their 2030 business-as-usual model using forecasts from either the national government or the IEA. Like the global business-as-usual model, they also made some large assumptions about the countries' future growth of GDP, population, and emissions. To develop the MACC from the business-as-usual model, McKinsey examined 120 to 200 greenhouse gas abatement opportunities for each country, usually across the five most important sectors–power, industry, transportation, buildings, and forestry/agriculture. McKinsey & Company assessed the technology levels within each sector and possible future advancements to appropriately adjust the business-as-usual model to forecast potential future emission abatement.

5.2 CO_2 Emission Levels and Population Data

Along with the MACCs, the fair division model requires more specific data for each country about their recent CO_2 emissions. Recent carbon emission data is used to find how much each country must abate to reach their initial emission permit entitlement. We assume that current emissions, c_i, are unrestricted, and can therefore be used to calculate each country's stand-alone utility, i.e. $v_i(\omega) = v_i(c_i)$. Also, we know the current emission level, c_i, less the abatement reduction at the global market price, a_i, is the final emission permit allocation, $c_i - a_i = x_i$. The most recent CO_2 emissions data by country is for 2007 and comes from the World Bank's *World Development Indicators*. We also gathered the CO_2 emission data for 1990 and population for 2007 for each country from the *World Development Indicators*, so we can "fairly" calculate initial emission permit entitlement, ω_i, for our simulation.

We found that 2007 emissions for all ten countries participating in our market simulation are about 17.2 gigatons of CO_2. Thus, the emissions of the ten countries within our simulation emitted the majority of 2007 global carbon emissions, about 27.2 gigatons of CO_2.

The first important calculation we must do is find the cap or number of emission permit, ω. As discussed in Section 2.2, 1990 emission levels are a desirable mile market for emission levels because it was the last time CO_2 concentrations were below 350 ppm, i.e., the safe upper limit for climate change. We will use the same cap as called for in the Kyoto Protocol, a 5.2% reduction below 1990 emission levels. Although the Protocol only applied this cap to Annex 1 countries, we are imposing it on everyone because as discussed earlier broad participation is extremely important. The total 1990 emission level for our ten countries is about 11.04 gigatons of CO_2. Thus, applying the 5.2% reduction,

$$\omega = 11.04 - 11.04(0.052) = 10.5$$

Thus, the cap is about 10.5 gigatons of CO_2. Since current emissions are about 17.2 gigatons CO_2 for our ten countries, we must reduce annual emissions by about 6.7 gigatons.

The market price of emission permits is where the marginal abatement costs for all ten countries are equal. To find this we used McKinsey's global MACC. Since we must reduce emission by about 6.7 gigatons of CO_2 per year, we find \$120 per ton of CO_2.

Now, that we have ω and p, we must find the initial entitlements, ω_i. We discussed two possible "fair" and justifiable rules for dividing up initial entitlements: 1) proportional to the 5.2% reduction of 1990 emission levels, or 2) per capita.

First, we discuss the proportional method. The proportional method is the more likely option out of these two because more countries will agree to it. The proportional method is based on the technique used in the Kyoto Protocol. Thus, initial entitlements are calculated from each country's 1990 carbon emission level, but with a 5.2% reduction. The difference from the Kyoto Protocol is that this reduction applies to all countries, not just developed or Annex 1 countries. To illustrate how the proportional method is calculated we provide an example below.

Example 5.2.0.1. (Calculating Proportional Initial Entitlements for Australia).
Refer to Table 7.2.4. Australia emitted 292.9 megatons of CO_2 in 1990. To find the proportional initial entitlement, ω_i, we calculate

$$\omega_{\text{Australia}} = 292.9 - 292.9(0.052) = 277.7$$

Thus, the Australia's proportional initial entitlement is about 277.7 megatons of CO_2.

Second, we discuss the per capita method. The per capita method gives a significant advantage to the world's most populous countries, China and India. The per capita method is not really a realistic option because the world's most powerful country, the United States, would never agree to it. It is important to include because the per capita market simulation will illustrate why the stand-alone upper bound is so important to the fair division model. To find the per capita initial entitlements, we first divide the total available amount of emission permits, ω, by the total population of all ten countries. The result is the amount of emission permits that each person receives. So to get the initial entitlement for each country, ω_i, we multiplied this result by the population of country i. To illustrate how the per capita method is calculated we provide an example below.

Example 5.2.0.2. (Calculating Per Capita Initial Entitlements for Australia).
From above, we know $\omega = 10,500$ megatons of CO_2. Refer to Table 7.2.4. The total population of all ten countries is 3,268,294,703. To find the emission permits per capita, we calculate

$$\frac{10,500}{3,268,294,703} = 0.0000032$$

Thus, each country will receive 3.2 emission permits for every 100,000 residents. Again, refer to Table 7.2.4. Australia's population is 21,874,900. So we then calculate

Australia's per capita intial entitlement by

$$\omega_{\text{Australia}} = 21,874,900(0.0000032) = 70$$

Thus, the Australia's per capita initial entitlement is about 70 megatons of CO_2.

5.3 Utility Functions and Calibration

Helm's model requires monotonic and quasilinear utility functions, $U_i(x_i, t_i) = v_i(x_i) + t_i$. But since agents are countries, instead of individuals, their utility functions are social welfare functions and therefore must represent the preferences of society. This may cause issues because most likely a certain level of pollution abatement for a whole country might affect different individuals or groups in that country differently. For example, an individual who commutes into the city for work will see the prices of gas increase, increasing his costs and therefore lowering his utility. On the other hand, an individual who lives in the city and rides his bike to work will have increased utility by the higher gas prices because there are less car on the road, so he does not have to worry about getting hit as much. This may be a problem for the Pareto efficiency criterion. Even though a BWS allocation is Pareto efficient for societal preferences, it may not be for individuals. We can argue against these criticisms of the countries' utility functions, if we assume that the individuals' utility functions are also monotonic and quasilinear, like the countries' utility functions. Since the individuals' utility functions are quasilinear, we know social welfare is maximized by choosing a level of pollution that maximizes the sum of everyone's utility, i.e. Proposition 4.3.1.6, but the agents are individuals, instead of countries. Thus, we can then transfer money between individuals to redistribute the utility in such a way that everyone is better off than they were originally, so the outcome is Pareto efficient for all individuals, as well as society. Whether this argument is true or not, to be confident in the use of the countries' utility functions, we assume that societal preferences arise from some political mechanism that aggregates individual preferences in a consistent manner over time.

Since we could not find suitable pollution utility functions for our ten countries, we must create them. The utility functions should be calculated in a manner that they are based on the country's abatement costs and its current emissions. A country's abatement costs, $MAC_i(a_i)$, are a function of abatement amounts, a_i, and come from the MACCs from McKinsey & Company.

We first decided that the utility curves should be strictly convex because it is an important assumption of the Walrasian equilibrium for Pareto efficiency. Differences among countries that make some more susceptible (like those in tropical areas) to climate change than other are considered irrelevant to the utility functions, because we assume the cap on emissions, ω, that we chose is low enough so these climate extremes will not occur. Therefore, we assume that all curves should have the same functional form of square root functions, $v_i(x_i) = z_i(x_i)^{\frac{1}{2}}$, where z_i represent the differences in preference for abatement costs and current emission between agents.

We calibrate z_i according to the countries' current level of emissions, c_i, and a particular abatement cost, $MAC_i(a_i)$. We chose the abatement cost to be \$120, i.e., $MAC_i(a_i) = 120$ for all $i \in I$. Using McKinsey & Company's MACCs, we can find a specific abatement amount, a_i, for each country that corresponds to the $MAC_i(a_i) = 120$.

Note 5.3.0.3. Note that: $c_i - a_i = x_i$.

We differentiate the utility curve and find that marginal utility is

$$Mv_i(x_i) = \frac{1}{2} z_i (x_i)^{-\frac{1}{2}}.$$

Since the marginal utility from polluting must be equal to the marginal savings from not having to abate, we know

$$\begin{aligned} MAC_i(a_i) &= Mv_i(x_i) \\ &= Mv_i(c_i - a_i) \\ &= \frac{1}{2} z_i (c_i - a_i)^{-\frac{1}{2}}. \end{aligned}$$

Thus, we can solve for z_i.

$$z_i = \frac{2MAC_i(a_i)}{(c_i - a_i)^{-\frac{1}{2}}} \tag{5.3.0.4}$$

$$= \frac{2(120)}{(c_i - a_i)^{-\frac{1}{2}}} \quad \text{since} \quad MAC_i(a_i) = 120. \tag{5.3.0.5}$$

We plug in our values for c_i and a_i into Equation (5.3.0.5) for each country to find their z_i's. Refer to Table 2 in the Appendix for the values of c_i, a_i, and z_i.

So we have created a calibrated utility curve for each country's utility function, i.e.,

$$U_i(x_i, t_i) = z_i (x_i)^{\frac{1}{2}} + t_i \quad \text{for all} \quad i \in I.$$

Chapter 6

The Market Simulations and Results

This chapter examines and analyzes the results of our two market simulations for proportional and per capita initial entitlements. We found some surprising results for proportional initial entitlements. Also, that the stand-alone upper bound is an important contstraint and the BWS final allocation is significantly different from the free market allocation in both simulations.

Remark 6.0.0.1. Note that by the quasilinearity of the utility functions, 1 utility point is equivalent to $1,000. These monetary figures are extremely sensitive to any changes in the MACCs, so we cannot place much reliance on them. But the results are still important because they help us understand each country's outcome in proportion to one another. We hope that McKinsey & Company calculated all of the MACCs in some consistent manner, so if some aspect of the forecasts is off, they will all be incorrect by the same amount.

6.1 Proportional Entitlements Market Simulation

The initial results from the emission permit market simulation with proportional initial entitlements are in Table 7.2.5. The results were quite surprising because we had expected that a free market with proportional initial entitlements would not violate the stand-alone upper bound. Five countries–Poland, Russia, Sweden, Switzerland, and the United Kingdom–all failed the minimum test for the free market as

$$v_i(\omega) - v_i(x_i) < t_{i,1} = p(\omega_i - x_i).$$

It occurred to us that maybe the stand-alone utilities for some of these countries are not correct because they have been participating in the European Union Emission Trading Scheme for the past three years and so their current emissions are not unrestricted, i.e., our assumption that $v_i(c_i) = v_i(\omega)$ fails. Since Poland, Sweden, and the United Kingdom all participate in the EU ETS, we altered their the stand-alone utilities to reflect 1990 emission levels, m_i, instead of current emissions. This means $v_i(m_i) = v_i(\omega)$ for Poland, Sweden, and the U.K. Table 7.2.6 is the 'fixed' results from the emission permit market simulation with proportional initial entitlements.

Even though we altered the stand-alone utilities of three countries, two countries–Switzerland and Russia–are still failing the minimum test. This is an interesting result because it was also unexpected. During our preliminary examination of Helm's model, we believed that the stand-alone upper bound would not cause significant differences to a traditional free market cap-and-trade scheme unless initial entitlements were allocated in a weird manner, like per capita or equally between countries. Clearly, we were wrong. Russia is significantly overcompensated in the free market for emission permits with a stand-alone violation that is about 27,000 utility points, which is equivalent to about $27 billion. Russia has such significant gains because initial entitlements are based on a 5.2% reduction of 1990 emission levels. Over the past twenty years, Russia has faced significant decreases in production, and therefore emissions, due to the collapse of the Soviet Union. Thus, they have plenty of extra emission permits to sell at a profit. Switzerland also violates the stand-alone upper bound, but at a much lesser extent, by about 500 utility points, which is equivalent to $500 million. Switzerland's current emissions are also lower than their 1990 levels, which can account for their overcompensation. Table 7.2.7 illustrates the redistribution of Russia and Switzerland's overcompensation to the unsatiated agents in the trading scheme. Each of the eight unsatiated agents receives some redistributed utility, $\lambda_1 = 3,443$, which is worth about $3.4 billion. Once this redistribution occurs, no country is overcompensated, and both Russia and Switzerland are receiving their stand-alone utilities.

6.2 Per Capita Initial Entitlements Market Simulation

The results from the emission permit market simulation with per capita initial entitlements are in Table 7.2.8. The results were along the lines that we expected with populous developing countries failing the minimum test, i.e. violating the stand-alone upper bound. Large developing countries violate the upper bound because they receive more initial entitlements than they need, and can therefore sell the excess for a large profit. On top of this, because these countries are developing they have lower marginal abatement costs than a lot of the other countries, and therefore choose to sell additional permits and abate, rather than pollute and keep the expensive permits. India and Brazil both initially violate the stand-alone constraint by a lot, about $265 billion and $37 billion, respectively. Because there is such a large violation amount, the redistribution to unsatiated agents is quite extensive with

$\lambda_1 = 37,722$. Refer to Table 7.2.9. When the unsatiated countries receive their equal redistribution amounts, two countries–Australia and Switzerland–are pushed over their stand-alone utility and now fail the minimum test. Therefore, another redistribution (Table 7.2.10) must take place giving $\lambda_2 = 6,598$ to the remaining unsatiated countries (note that India and Brazil are both receiving their stand-alone utility and are therefore satiated, but not violating the upper bound). This is the last redistribution because no countries fail the minimum test, i.e. violate the stand-alone upper bound.

In both market simulations, the emission trading market is worth about $1.88 trillion and the countries always end up with the same final allocation of permits. The difference is how the costs are divided among countries. It appears that the proportional initial entitlement is a more "fair" method of initially allocation the emission permits because the burden of costs are spread more equally across countries than with the per capita initial entitlements. Three countries–Australia, China and India–are buying permits and the rest of the countries are selling them. India is buying the bulk of the permits worth about $63 billion. India must buy so many emission permits because the country has experienced major growth in the last twenty years, and therefore increased their emissions significantly since 1990. The per capita initial entitlements force the burden of the costs onto Russia and the United States, paying about $52 billion and $286 billion, respectively, for their additional emission permits. Politically, proportional initial entitlements are more feasible because more countries would be willing to participate in the trading scheme, where as with per capita the United States would definitely opt out as they are burdened with the majority of the costs.

Chapter 7

Conclusion

This paper explores an alternative cap-and-trade system proposed by Helm (2008) that uses four properties of fair division–Pareto efficiency, individual rationality, stand-alone upper bound, and envy-freeness with respect to a compensation rule–to guarantee "fairness." We illustrated possible outcomes of this fair division emission trading scheme, but simulating the emission trading market using a subset of ten countries. We used marginal abatement cost curves from McKinsey & Company and recent emission data from the *World Development Indictors* to reasonably mimic what results that would occur in a global emission trading scheme. We found the surprising result that the incorporation of the stand-alone upper bound into the Walrasian solution leads to a much different market outcome, than a free market cap-and-trade scheme. The stand-alone upper bound may be a great addition to any cap-and-trade scheme to guarantee that no countries are overcompensated within the trading system.

There is no easy solution to global climate change. Whatever form of mitigation that the United Nations eventually agrees on (if ever) must appease the vast majority of the countries participating, including the most powerful–the United States and

China. Although Helm's scheme presents some promising results when it comes to efficiency and envy-freeness, we find that the most important aspect of equal burden sharing is how the initial entitlements are divided up. Helm supposes that entitlements are divided in some "equal" manner, but does not specify further. In this paper, I proposed possible justifiable methods of dividing up initial entitlements–proportional based on 1990 emissions and per capita. It is evident from the results that that neither of these methods are satisfactory because the burden sharing is not divided up in a logical manner. A suggestion for further research is exploring more possible justifiable methods for dividing up initial entitlements, like a proportional reduction from current emissions or an equal division between all countries.

Appendix

7.1 Model Appendix

Utility Functions 7.1.1. The Utility Functions for the Agents in Example 4.3.4.5:

$$U_1(x_1, t_1) = \frac{12x_1}{x_1 + 4} \qquad U_2(x_2, t_2) = \frac{8x_2}{x_2 + 2}$$

$$U_3(x_3, t_3) = \frac{12x_3}{x_3 + 4} \qquad U_4(x_4, t_4) = \frac{10x_4}{x_4 + 3}$$

7.2 Figure and Tables Appendix

Figure 7.2.1: Russia's Marginal Abatement Cost Curve for 2030 [McKinsey]

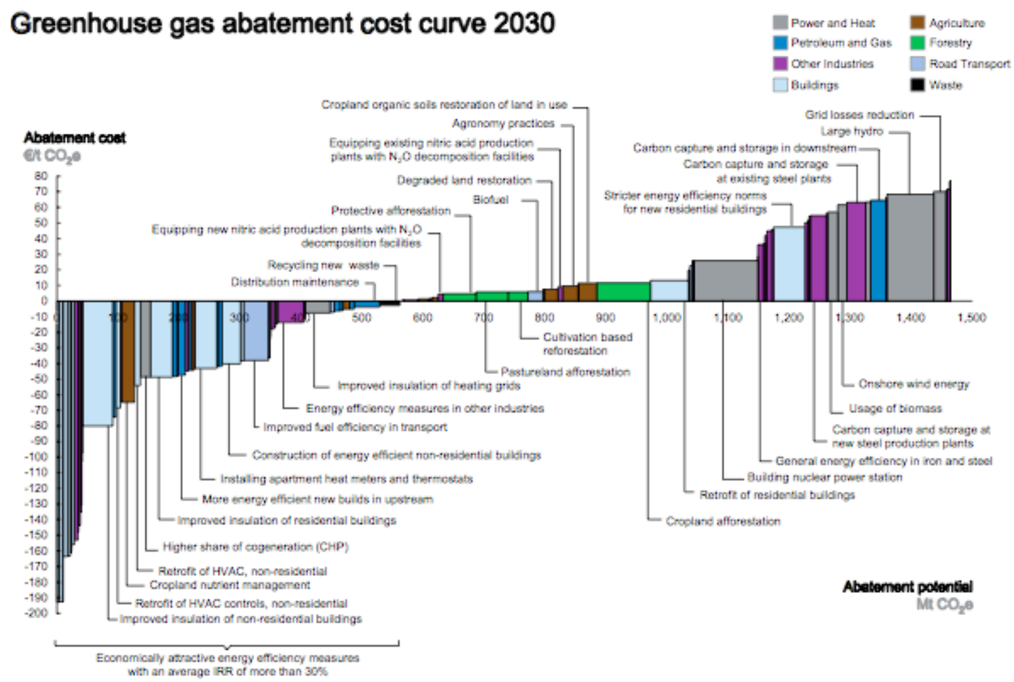

Table 7.2.1: Step 1 from Example 4.3.4.5

Free Market:

Agents	ω_i	$v_i(\omega)$	$v_i(\omega_i)$	x_i	$v_i(x_i)$	$v_i(\omega) - v_i(x_i)$	$p(\omega_i - x_i)$	$t_{i,1}$	Pass Min. Test?	Overcomp.
1	26	11.76	10.4	59.24	11.24	0.52	-0.40	-0.40	Yes	0
2	70	7.92	7.78	34.51	7.56	0.36	0.42	0.42	NO!!	0.07
3	20	11.76	10	59.25	11.24	0.52	-0.47	-0.47	Yes	0
4	84	9.85	9.66	47.00	9.40	0.45	0.44	0.44	Yes	0

Table 7.2.2: Step 2 from Example 4.3.4.5

Redistributed Market from Agent 2:

Agents	ω_i	$v_i(\omega)$	$v_i(\omega_i)$	x_i	$v_i(x_i)$	$v_i(\omega) - v_i(x_i)$	$p(\omega_i - x_i) + \lambda_1$	$t_{i,2}$	Pass Min. Test?	Overcomp.
1	26	11.76	10.4	59.24	11.24	0.52	-0.38	-0.38	Yes	0
2	70	7.92	7.78	34.51	7.56	0.36	0.45	0.36	Yes	0
3	20	11.76	10	59.25	11.24	0.52	-0.45	-0.45	Yes	0
4	84	9.85	9.66	47.00	9.40	0.45	0.47	0.47	NO!!	0.02

Table 7.2.3: Step 3 from Example 4.3.4.5

Redistributed Market from Agent 4:

Agents	ω_i	$v_i(\omega)$	$v_i(\omega_i)$	x_i	$v_i(x_i)$	$v_i(\omega) - v_i(x_i)$	$p(\omega_i - x_i) + \lambda_2$	$t_{i,3}$	Pass Min. Test?	Overcomp.
1	26	11.76	10.4	59.24	11.24	0.52	-0.37	-0.37	Yes	0
2	70	7.92	7.78	34.51	7.56	0.36	0.46	0.36	Yes	0
3	20	11.76	10	59.25	11.24	0.52	-0.44	-0.44	Yes	0
4	84	9.85	9.66	47.00	9.40	0.45	0.47	0.45	Yes	0

Table 7.2.4: Summary of the Data

Countries	Current (2007) Emissions (Mt), c_i	1990 Emission Levels	5.2% Reduction of 1990 Levels (Proportional Initial Entitlement), w_i	Population	Per Capita Initial Entitlement, w_i	Amount of Abatement, a_i	z_i
Australia	373.74	292.88	277.65	21,874,900	70.05	47	3,253.57
Brazil	368.02	208.72	197.86	193,733,795	620.42	150	2,657.79
China	6,533.02	2,458.73	2,330.88	1,331,460,000	4,263.94	4,160	8,768.46
India	1,611.04	690.01	654.13	1,155,347,678	3,699.94	400	6,264.00
Poland	317.12	347.57	329.50	38,149,886	122.17	15	3,128.69
Russia	1,536.10	1,847.29	1,751.23	141,850,000	454.27	40	6,962.30
Sweden	49.21	80.76	76.56	9,302,123	29.79	1	1,249.80
Switzerland	37.96	39.07	37.03	7,731,167	24.76	15	862.50
United Kingdom	539.18	603.15	571.79	61,838,154	198.03	53	3,968.91
United States	5,832.19	4,472.49	4,239.92	307,007,000	983.17	1,850	11,358.83
Total	17,197.57	11,040.67	10,466.55	3,268,294,703	10,466.55		

Table 7.2.5: Free Market for Proportional Initial Entitlements

Countries	ω_i	$v_i(\omega)$	$v_i(\omega_i)$	x_i	$v_i(x_i)$	$v_i(\omega) - v_i(x_i)$	$p(\omega_i - x_i)$	$t_{i,1}$	Pass Min. Test?	Overcomp.
Australia	62,897.52	277.66	54,214.21	326.72	58,809.60	4,087.92	-5,887.80	-5,887.80	Yes	0
Brazil	50,986.64	197.86	37,385.20	218.02	39,243.60	11,743.04	-2,419.20	-2,419.20	Yes	0
China	708,728.99	2,330.89	423,334.92	2,373.02	427,143.60	281,585.39	-5,055.60	-5,055.60	Yes	0
India	251,422.95	654.13	160,207.85	1,211.04	217,987.20	33,435.75	-66,829.20	-66,829.20	Yes	0
Poland	55,715.25	329.50	56,792.36	302.12	54,381.60	1,333.65	3,285.60	3,285.60	NO!!	1,951.95
Russia	272,874.25	1,751.23	291,356.30	1,496.10	269,298.00	3,576.25	30,615.60	30,615.60	NO!!	27,039.35
Sweden	8,767.34	76.56	10,935.59	48.21	8,677.80	89.54	3,402.00	3,402.00	NO!!	3,312.46
Switzerland	5,314.00	37.03	5,248.50	22.96	4,132.80	1,181.20	1,688.40	1,688.40	NO!!	507.20
United Kingdom	92,159.04	571.79	94,905.05	486.18	87,512.40	4,646.64	10,273.20	10,273.20	NO!!	5,626.56
United States	867,459.74	4,239.92	739,626.26	3,982.19	716,794.20	150,665.54	30,927.60	30,927.60	Yes	0

Table 7.2.6: Free Market (fixed) for Proportional Initial Entitlements

Countries	ω_i	$v_i(\omega)$	$v_i(\omega_i)$	x_i	$v_i(x_i)$	$v_i(\omega) - v_i(x_i)$	$p(\omega_i - x_i)$	$t_{i,1}$	Pass Min. Test?	Overcomp.
Australia	62,897.52	277.66	54,214.21	326.72	58,809.60	4,087.92	-5,887.80	-5,887.80	Yes	0
Brazil	50,986.64	197.86	37,385.20	218.02	39,243.60	11,743.04	-2,419.20	-2,419.20	Yes	0
China	708,728.99	2,330.89	423,334.92	2,373.02	427,143.60	281,585.39	-5,055.60	-5,055.60	Yes	0
India	251,422.95	654.13	160,207.85	1,211.04	217,987.20	33,435.75	-66,829.20	-66,829.20	Yes	0
Poland	1,844,531.66	329.50	56,792.36	302.12	54,381.60	1,790,150.06	3,285.60	3,285.60	Yes	0
Russia	272,874.25	1,751.23	291,356.30	1,496.10	269,298.00	3,576.25	30,615.60	30,615.60	NO!!	27,039.35
Sweden	355,168.51	76.56	10,935.59	48.21	8,677.80	346,490.71	3,402.00	3,402.00	Yes	0
Switzerland	5,314.00	37.03	5,248.50	22.96	4,132.80	1,181.20	1,688.40	1,688.40	NO!!	507.2
United Kingdom	3,082,360.89	571.79	94,905.05	486.18	87,512.40	2,994,848.49	10,273.20	10,273.20	Yes	0
United States	867,459.74	4,239.92	739,626.26	3,982.19	716,794.20	150,665.54	30,927.60	30,927.60	Yes	0

Table 7.2.7: Redistribution Market from Table 7.2.6 (redistribution from Russia & Switzerland)

Countries	ω_i	$v_i(\omega)$	$v_i(\omega_i)$	x_i	$v_i(x_i)$	$v_i(\omega) - v_i(x_i)$	$p(\omega_i - x_i) + \lambda_1$	$t_{i,2}$	Pass Min. Test?	Overcomp.
Australia	62,897.52	277.66	54,214.21	326.72	58,809.60	4,087.92	-2,444.48	-2,444.48	Yes	0
Brazil	50,986.64	197.86	37,385.20	218.02	39,243.60	11,743.04	1,024.12	1,024.12	Yes	0
China	708,728.99	2,330.89	423,334.92	2,373.02	427,143.60	281,585.39	-1,612.28	-1,612.28	Yes	0
India	251,422.95	654.13	160,207.85	1,211.04	217,987.20	33,435.75	-63,385.88	-63,385.88	Yes	0
Poland	1,844,531.66	329.50	56,792.36	302.12	54,381.60	1,790,150.06	6,728.92	6,728.92	Yes	0
Russia	272,874.25	1,751.23	291,356.30	1,496.10	269,298.00	3,576.25	34,058.92	3,576.25	Yes	0
Sweden	355,168.51	76.56	10,935.59	48.21	8,677.80	346,490.71	6,845.32	6,845.32	Yes	0
Switzerland	5,314.00	37.03	5,248.50	22.96	4,132.80	1,181.20	5,131.72	1,181.20	Yes	0
United Kingdom	3,082,360.89	571.79	94,905.05	486.18	87,512.40	2,994,848.49	13,716.52	13,716.52	Yes	0
United States	867,459.74	4,239.92	739,626.26	3,982.19	716,794.20	150,665.54	34,370.92	34,370.92	Yes	0

Table 7.2.8: Free Market (fixed) for Per Capita Initial Entitlements

Countries	ω_i	$v_i(\omega)$	$v_i(\omega_i)$	x_i	$v_i(x_i)$	$v_i(\omega) - v_i(x_i)$	$p(\omega_i - x_i)$	$t_{i,1}$	Pass Min. Test?	Overcomp.
Australia	62,897.52	70.05	27,231.69	326.72	58,809.60	4,087.92	-30,800.00	-30,800.00	Yes	0
Brazil	50,986.64	620.42	66,201.01	218.02	39,243.60	11,743.04	48,288.36	48,288.36	NO!!	36,545.32
China	708,728.99	4,263.94	572,569.90	2,373.02	427,143.60	281,585.39	226,909.85	226,909.85	Yes	0
India	251,422.95	3,699.94	381,021.38	1,211.04	217,987.20	33,435.75	298,668.52	298,668.52	NO!!	265,232.78
Poland	1,844,531.66	122.17	34,582.00	302.12	54,381.60	1,790,150.06	-21,593.62	-21,593.62	Yes	0
Russia	272,874.25	454.27	148,391.40	1,496.10	269,298.00	3,576.25	-125,019.88	-125,019.88	Yes	0
Sweden	355,168.51	29.79	6,821.40	48.21	8,677.80	346,490.71	-2,210.45	-2,210.45	Yes	0
Switzerland	5,314.00	24.76	4,291.63	22.96	4,132.80	1,181.20	215.84	215.84	Yes	0
United Kingdom	3,082,360.89	198.03	55,852.24	486.18	87,512.40	2,994,848.49	-34,577.56	-34,577.56	Yes	0
United States	867,459.74	983.17	356,163.07	3,982.19	716,794.20	150,665.54	-359,881.81	-359,881.81	Yes	0

Table 7.2.9: Redistribution Market from Table 7.2.9 (redistribution from Brazil & India)

Countries	ω_i	$v_i(\omega)$	$v_i(\omega_i)$	x_i	$v_i(x_i)$	$v_i(\omega) - v_i(x_i)$	$p(\omega_i - x_i) + \lambda_1$	$t_{i,2}$	Pass Min. Test?	Overcomp.
Australia	62,897.52	70.05	27,231.69	326.72	58,809.60	4,087.92	6,922.26	6,922.26	NO!!	2,834.34
Brazil	50,986.64	620.42	66,201.01	218.02	39,243.60	11,743.04	86,010.62	11,743.04	Yes	0
China	708,728.99	4,263.94	572,569.90	2,373.02	427,143.60	281,585.39	264,632.12	264,632.12	Yes	0
India	251,422.95	3,699.94	381,021.38	1,211.04	217,987.20	33,435.75	336,390.79	33,435.75	Yes	0
Poland	1,844,531.66	122.17	34,582.00	302.12	54,381.60	1,790,150.06	16,128.64	16,128.64	Yes	0
Russia	272,874.25	454.27	148,391.40	1,496.10	269,298.00	3,576.25	-87,297.62	-87,297.62	Yes	0
Sweden	355,168.51	29.79	6,821.40	48.21	8,677.80	346,490.71	35,511.81	35,511.81	Yes	0
Switzerland	5,314.00	24.76	4,291.63	22.96	4,132.80	1,181.20	37,938.10	37,938.10	NO!!	36,756.90
United Kingdom	3,082,360.89	198.03	55,852.24	486.18	87,512.40	2,994,848.49	3,144.70	3,144.70	Yes	0
United States	867,459.74	983.17	356,163.07	3,982.19	716,794.20	150,665.54	-322,159.55	-322,159.55	Yes	0

Table 7.2.10: Redistribution Market from Table 7.2.10 (redistribution from Australia & Switzerland)

Countries	ω_i	$v_i(\omega)$	$v_i(\omega_i)$	x_i	$v_i(x_i)$	$v_i(\omega) - v_i(x_i)$	$p(\omega_i - x_i) + \lambda_2$	$t_{i,3}$	Pass Min. Test?	Overcomp.
Australia	62,897.52	70.05	27,231.69	326.72	58,809.60	4,087.92	13,520.80	4,087.92	Yes	0
Brazil	50,986.64	620.42	66,201.01	218.02	39,243.60	11,743.04	92,609.16	11,743.04	Yes	0
China	708,728.99	4,263.94	572,569.90	2,373.02	427,143.60	281,585.39	271,230.66	271,230.66	Yes	0
India	251,422.95	3,699.94	381,021.38	1,211.04	217,987.20	33,435.75	342,989.33	33,435.75	Yes	0
Poland	1,844,531.66	122.17	34,582.00	302.12	54,381.60	1,790,150.06	22,727.18	22,727.18	Yes	0
Russia	272,874.25	454.27	148,391.40	1,496.10	269,298.00	3,576.25	-80,699.08	-80,699.08	Yes	0
Sweden	355,168.51	29.79	6,821.40	48.21	8,677.80	346,490.71	42,110.35	42,110.35	Yes	0
Switzerland	5,314.00	24.76	4,291.63	22.96	4,132.80	1,181.20	44,536.64	1,181.20	Yes	0
United Kingdom	3,082,360.89	198.03	55,852.24	486.18	87,512.40	2,994,848.49	9,743.24	9,743.24	Yes	0
United States	867,459.74	983.17	356,163.07	3,982.19	716,794.20	150,665.54	-315,561.01	-315,561.01	Yes	0

Bibliography

Ackerman, Frank, et al. 2009. "The Economics of 350: The Benefits and Costs of Climate Stabilization." Economics for Equity and the Environment Network.

Aldy, Joseph E., Eduardo Ley, and Ian W. H. Parry. 2008. "A Tax-Based Approach to Slowing Global Climate Change." Resources for the Future.

Arrow, Kenneth and Gerard Debreu. 1954. "Existence of an Equilibrium for a Competitive Economy." 22: 265–290.

Bhatti, Yosef, Kasper Lindskow, and Lene Holm Pedersen. 2010. "Burden-Sharing and Global Climate Negotiations: The Case of the Kyoto Protocol." Climate Policy 10, no. 2: 131–147.

Carlen, Bjorn. 2003. "Market Power in International Carbon Emissions Trading: A Laboratory Test." Energy Journal 24, no. 3: 1-26.

Daskalakis, George, and Raphael N. Markellos. 2008. "Are the European Carbon Markets Efficient?" Review of Futures Markets 17, no. 2: 103–128.

Dowdey, Sarah. 2007. "How Carbon Tax Works." HowStuffWorks.com. <http://science.howstuffworks.com/environmental/green-science/carbon-tax.htm>

Ehrhart, Karl-Martin, Christian Hoppe, and Ralf Loschel. 2008. "Abuse of EU Emissions Trading for Tacit Collusion." Environmental and Resource Economics 41, no. 3: 347–361.

Genesis 13: 6-9. New American Standard Bible.

Gerlagh, Reyer, and Bob van der Zwaan. 2006. "Options and Instruments for a Deep Cut in CO2 Emissions: Carbon Dioxide Capture or Renewables, Taxes or Subsidies?" Energy Journal 27, no. 3: 25–48.

Hagem, Cathrine. 2009. "The Clean Development Mechanism versus International Permit Trading: The Effect on Technological Change." Resource and Energy Economics 31, no. 1: 1-12.

Harrington, Winston and Richard Morgenstern. 2004. "Economic Incentives versus Command and Control." Resources 152: 13–17.

Helm, Carsten. 2008. "Fair division theory and climate change policy," Environmental and Development Economics, 13: 441–455.

Intergovernmental Panel on Climate Change (IPCC). 2007. "Climate Change 2007: Synthesis Report." Valencia, Spain: Intergovernmental Panel on Climate Change.

Kennedy, Peter W. and Benoit Laplante. 1999. "Environmental Policy and Time Consistency: Emissions Taxes and Emissions Trading." The World Bank, Policy Research Working Paper Series: 2351.

Martin, Will. 1999. "Reducing Carbon Dioxide Emissions through Joint Implementation of Projects." The World Bank, Policy Research Working Paper Series: 2359.

McKinsey & Company. 2009. "Pathways to a Low-Carbon Economy: Version 2 of the Global Greenhouse Gas Abatement Cost Curve." <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2008. "An Australian Cost Curve for Greenhouse Gas Reduction." <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2009. "China's Green Revolution: Prioritizing technologies to achieve energy and environmental sustainability." <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2009. "Pathways to a Low-Carbon Economy in Brazil." <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2009. "Swiss greenhouse gas abatement cost curve." <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2008. "Greenhouse gas abatement opportunities in Sweden." <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2007. "Reducing U.S. greenhouse gas emissions: How much at what cost?." <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2007. "Climate change: Everyone's business–options for greenhouse gas reduction in the UK." <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2009. "Greenhouse gas abatement potential in Poland." <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2009. "Pathways to an energy- and carbon-efficient Russia."

<http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

McKinsey & Company. 2009. “Environmental and energy sustainability: An approach for India.” <http://www.mckinsey.com/clientservice/sustainability/Costcurves.asp>

Nordhaus, William D. 2008. A Question of Balance: Weighing the Options on Global Warming Policies. New Haven: Yale University Press.

Nordhaus, William D. 1993. “Rolling the ’DICE’: An Optimal Transition Path for Controlling Greenhouse Gases.” Resource and Energy Economics 15, no. 1: 27–50.

Robertson, Jack and William Webb. Cake-Cutting Algorithms: Be Fair If You Can. Massachusetts: A K Peters.

Schreuder, Yda. 2009. The Corporate Greenhouse: Climate Change Policy and Greenhouse Gas Emissions. London: Zed Books.

Stern, Nicholas. 2007. The Economics of Climate Change: The Stern Review. New York: Cambridge University Press.

Stranlund, John, James Murphy, and John Spraggon. 2008. “Imperfect Enforcement of Emissions Trading and Industry Welfare: A Laboratory Investigation.” UMass Amherst Department of Resource Economics Working Paper: No. 2008-1.

World Bank. 2011. “CO_2 Emissions (kt).” World Development Indicators. <http://data.worldbank.org/indicator/EN.ATM.CO2E.KT>

350.org. 2011. “The Science of 350.” 350.org. <http://www.350.org/en/about/science>