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Accurately Simulating the Battle of Thermopylae to Analyze "What If" Scenarios

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By

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Abstract

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The Battle of Thermopylae (480 BCE) was a last ditch effort to stall the Persian army as it marched south toward Athens. Led by Leonidas and his personal guard of 300 Spartans, a citizen army of Greeks was able to delay a Persian army of over 100,000 soldiers at the town of Thermopylae for several days. Although the Greeks were ultimately defeated at Thermopylae, the battle provided enough time for the Greek states to regroup and plan a counter attack, eventually defeating the invading Persians. This battle was crucial not only for the preservation of Greek independence, but also the first democratic society. But how, against all odds, did the small Greek force withstand the Persians for as long as they did? What might have happened had key factors and choices been slightly different? By modeling the battle with historical accuracy, we can simulate various counter-factual scenarios to determine which factors were crucial for the Greeks' incredible achievement.

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1 Introduction

The Battle of Thermopylae, 480 BCE, is regarded as one of the most iconic battles in ancient history. The battle is most notable for the unbelievable resilience shown by a few thousand Greek soldiers in the face of certain death. Their foe, the Persians, had over 100,000 soldiers in their army and were advancing south, toward Athens, with the goal of destroying the Greek state.[1] Although they were ultimately defeated, the Greeks were able to stall the Persians at Thermopylae for three days, giving the Greeks enough time to evacuate Athens and plan a counter attack, ultimately pushing the Persians out of Greece. In doing so, the Greeks protected the Athenian democracy, preserving it for future civilizations.

In retrospect, there is absolutely no doubt about the important role that this battle had on the course of history. However, this acknowledgement does not help to explain how significant the specific circumstances of the battle were to this outcome. For centuries, scholars produced countless hypotheses and theories that claimed to explain the details of this battle and other historical events. But, they were restrained by the lack of knowledge on the subject since Herodotus's account of the battle remains its sole primary source. As a result, there was a very small amount of new knowledge being discovered about the battle and other historical subjects, as most scholars made simple speculations or wildly unsupported conclusions. However, a sudden interest in the ancient world over the last century spurred a new wave of scholarship that uses modern technology as a means to discover new information about these important events.

With the onset of modern archaeology at the end of the 19th century, the study of ancient history was introduced to the scientific method.[2] For the first time in over a millennium, scholars began to search for the remnants of ancient history, using ancient authors like Herodotus and Homer as their guides. With the discovery of these ancient sites, new sources of knowledge were finally unearthed. These discoveries led to a major shift in the study of ancient history, as scholars could compare archaeological evidence with the historical records. Saturated by an overwhelming amount of scholarship, this new wave of interest in historical analysis slowly subsided. Suddenly, the discipline reached a new intellectual horizon that resulted in the publication of more wild theories and unsupported conclusions. This time, scholars tried to link archaeological discoveries with historical records but could not provide any evidence to prove that their conclusions were accurate. They lacked an important link between these two sources that made it impossible to conclude, with complete certainty, anything pertaining to the way the ancient world worked or the accuracy of the historical record.

However, Peter Turchin, a professor from the University of Connecticut, believed that he could solve this issue by using analytic models generated by computer simulations in an interdisciplinary field he

[1] Herodotus. *The Histories*. Trans. by Tom Holland and Paul Cartledge. New York: Penguin Group, 2014, 7.60.
[2] William H. Stiebing. *Uncovering the Past. A History of Archaeology*. Oxford University Press, 1994.

called *Cliodynamics*.[3] Believing that truly understanding the present requires an understanding of the past, Turchin began to study and analyze history using complex simulations. These simulations modeled the interactions between individual agents as a means to uncover the factors that led to the rise and fall of great civilizations.[4] Using Turchin's method, unsupported theories can be tested and analyzed, leading to important conclusions that may unveil new historical knowledge. This approach has been used to conduct research in a variety of interesting studies: from explaining the settlement patterns of the Incan Empire around Lake Titicaca in South America,[5] to populating the Roman Colosseum with virtual crowds.[6]

Motivated by these cliodynamic methods, I developed a simulation of the Battle of Thermopylae that I used to analyze "What-if" scenarios. By developing a historical model that produced historically accurate simulation results, I could confidently simulate counter-factual scenarios that altered the specific circumstances of the battle. Analyzing the differences between the results of these counter-factual simulations and the historical simulations revealed the significant impact these details had on the outcome of the battle.

2 Background and Related Work

2.1 Cliodynamics

This interdisciplinary approach to analyzing history originated with the concept of *cliodynamics*, developed by Peter Turchin at the University of Connecticut.[7] Turchin argues that although we put forth a massive amount of resources into research that will strengthen and fix our sickly society, we are inherently disadvantaged by our lack of understanding of how we came to be sick in the first place. He proposes that it is time to treat history as "an analytic, and even predictive, science," and suggests that we pursue the new interdisciplinary field of *cliodynamics*. This discipline marries the study of history with the social sciences in order to develop theories that explain, with an analytic perspective, why and how a society developed certain problems. By testing these theories using historical data and analytical techniques, we can make conclusions about the origins of society's greatest problems and begin to develop ways to correct them.

In one of his most recent publications, Turchin[8] studied the completeness of theories that "explain the emergence of large-scale societies." They realized that a problem with many of these theories was that they failed to explain how societies dealt with other cooperative groups. Additionally, most of these theories are

[3] Peter Turchin. "Arise 'cliodynamics'". In: *Nature: International Weekly Journal of Science* 454 (2008).

[4] Peter Turchin et al. "War, space, and the evolution of Old World complex societies". In: *Proceedings of the National Academy of Sciences of the United States of America* 110 (2013).

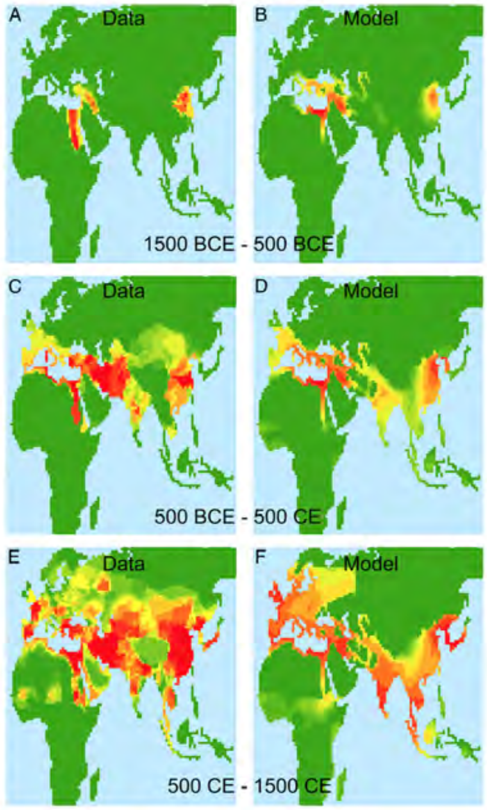
[5] Arthur Griffin and Charles Stanish. "An Agent-based Model of Prehistoric Settlement Patterns and Political Consolidation in the Lake Titicaca Basin of Peru and Bolivia". In: *Structure and Dynamics: eJournal of Anthropological and Related Sciences* 2.2 (2007), p. 49.

[6] Diego Gutierrez et al. "AI and virtual crowds: Populating the Colosseum". In: *Journal of Cultural Heritage* 8 (2007).

[7] Turchin, "Arise 'cliodynamics'".

[8] Turchin et al., "War, space, and the evolution of Old World complex societies".

Figure 1: Models A,C, and E represent the actual distribution of large communities in Afro-eurasia between 1500 BCE and 1500 CE. Models B, D, and F, are those made from the agent-based simulation.



verbal, leading to models that are non-explicit and barely proven. Turchin set out to test these theories with models made from agent-based simulations. This agent-based solution turned the proposed theory into a quantitative model. The result, as seen in Figure 1, was proof that their simulation was accurate. The simulation produced a model that predicted the distribution of large societies that was incredibly similar to the historical distribution. Using this simulator as the foundation, Turchin et al. proved that their distribution theory produced models that accurately generated the actual, historical distribution a significant portion of the time. They concluded that their results were an important step in developing the world of analytical history.

2.2 Using artificial intelligence in virtual environments

In a recent study, autonomous agents were used to simulate crowds populating the Roman Colosseum.[9] They argue that by using artificial intelligence in a historically accurate virtual environment, scholars are able to make conclusions about unknown details of history. This is made possible because of the predictive power of this type of autonomous simulation in a virtual environment. The simulation was built using a 3-dimensional model of the Colosseum from a different study. In order to populate this 3-dimensional virtual

[9] Gutierrez et al., "AI and virtual crowds: Populating the Colosseum".

reconstruction they needed to create intelligent crowds that could interact with this environment. They built an artificial intelligence system that was "not deterministic," meaning that its results are not scripted. By creating virtual agents with a sophisticated basic structure, Gutierrez et al. were able to build crowds that had senses, intelligence, knowledge, perception, goals, and predefined behaviors. Using a motor, these individual agents could act appropriately within the environment.[10] With so many possible behaviors, the agents used a finite state machine which defined the appropriate behaviors for every possible state. A finite state machine uses a series of sensors to receive environmental data, the interpretation of which changes the state to correspond with the current environment.[11] This finite state system organizes the agent's actions into generic behavioral groups, allowing them to act autonomously within the bounds of the program. With all of these components working together, crowds of artificially intelligent agents are able to interact with the virtual environment freely, generating unpredictable results. Analyzing these results can lead to new insight and logical conclusions about how people interacted with these historical environments.

3 The History

By the early 5th century BCE, the Persian Empire, under King Darius I (r.522-486 BCE), controlled the territory between the Persian Gulf and modern day Greece (Figure 2).[12] As the empire expanded, Greeks living in newly occupied territories in Asia Minor were given the opportunity to submit to the Persians without consequence. This meant accepting Persian appointed tyrants as local leaders, economic blockades and a loss of liberty.[13] Many small city-states had no choice but to give in to Darius's demands. In some cases, however, the Greeks opted to resist the occupation. In Lydia, a coastal region of Asia Minor, many Greeks opposed Persian rule and planned to revolt. Unfortunately, a large portion of Lydians did not support the movement and the Persian army easily maintained control of the region. Several years later, Lydians had come to hate their Persian leaders and again planned a revolution. This time, they requested support from the mainland Greeks. In response, the Athenians sent soldiers and supplies to aid in the revolt. The Ionian Revolution, as it is now called, started in 499 BCE. With Athenian aid, the Ionian army defeated the Persians in Asia Minor with ease. However, once Darius learned of his defeat, he ordered his entire army to retake their lost territory and set his sights on mainland Greece.[14]

After unsuccessfully extending his offer of peaceful surrender to the mainland Greeks, Darius and his

[10] Gutierrez et al., "AI and virtual crowds: Populating the Colosseum", p. 9.
[11] Ulrich Nehmzow. *Mobile Robotics. A Practical Introduction*. 2nd ed. Springer-Verlag London, 2003.
[12] Adrian Bivar and Mark J. Dresden. "Ancient Iran". In: *Encyclopedia Britannica*. Encyclopedia Britannica, inc., 2009. URL: https://www.britannica.com/place/ancient-Iran.
[13] Ernle Bradford. *Thermopylae. The Battle for the West*. Open Road Media, 2014, p. 34.
[14] Nic Fields and Steve Noon. *Thermopylae 480 BC: Last Stand of the 300*. Campaign. Osprey Publishing, 2007, p. 9.

Figure 2: The extent of the Achaemenid (Persian) Empire circa 500 BCE. https://www.britannica.com/place/ancient-Iran

army set sail across the Aegean Sea. Aware of Darius's intentions, the Athenians and their allies prepared to fight near the town of Marathon. Here, in 490 BCE, the Persian army disembarked their ships and were quickly dispatched by the Greek army. After their defeat at the Battle of Marathon, the Persians retreated back across the Aegean Sea.[15] For 10 years, the Greeks celebrated their unlikely victory by erecting huge monuments dedicated to their gods. During this same period, however, the Persian Empire underwent massive changes when King Darius I died and his successor, Xerxes, again set his sights on Greece. Learning from Darius's mistakes, Xerxes planned to accompany his navy with a ground invasion. With over 100,000 soldiers in their army, the Persians marched into Greece with minimal resistance. Aware that Xerxes would quickly make his way to Athens, the Greeks had no choice but to stall the invasion in order to evacuate major cities, regroup, and devise a plan of action. The Greeks determined that the only way they could slow the Persians' swift march was to confront the Persian army on two fronts: by sea, at Artemisium, and by land, at Thermopylae (Figure 3).[16]

3.1 The Battle of Thermopylae

The Greek army sent to Thermopylae was led by the Spartan King Leonidas. Although Sparta had between 8,000 and 10,000[17] battle-ready soldiers at its disposal, the call to arms coincided with the Festival of Apollo Carnea, the celebration of the third moon of Fall. As religious people, the Spartans were not allowed to participate in battle during the festival.[18] However, to support the Greek cause and inspire their allies to

[15] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 10.

[16] United States Military Academy, West Point. *Atlas for Ancient Warfare: The Battle of Thermopylae*. URL: `http://www.usma.edu/history/SitePages/AncientWarfare`.

[17] Herodotus, *The Histories*, 7.234.

[18] Herodotus, *The Histories*, 7.206.

Figure 3: The locations of the Battles of Thermopylae and Artemisium. Map from the Department of History at the United States Military Academy, West Point.

join the war effort, the Spartans sent Leonidas and 300 hand-picked soldiers as an advance guard to Thermopylae. Herodotus mentions that although the rest of the Spartan army would march to Thermopylae after the festival, the soldiers who accompanied Leonidas were decidedly those who had already produced male heirs in case they died before the main army arrived.[19]

As the Spartan advance guard marched toward Thermopylae, they were joined by soldiers from neighboring city-states who had also answered the call to arms. By the time Leonidas arrived at Thermopylae, his advanced guard had grown to roughly 5,200 soldiers from all over Greece.[20] However, the Greeks were still at a significant disadvantage as, according to Herodotus, Xerxes had an army of more than two million soldiers.[21] Although modern scholars speculate about the accuracy of this claim suggesting that the army was at most 100,000 strong, the point remains the same.[22] Maximizing the effectiveness of his small force, Leonidas set up a defensive position at a place known as the "Middle Gate" in the region of Thermopylae (Figure 4).[23] Here, the coastal road passed through a natural bottle neck that was only 15 meters wide, flanked by steep mountain cliffs to the south and the sea to the north. By the Greeks defending this narrow section of the pass, the Persians would be forced to use smaller waves of soldiers against the powerful Greek phalanx, nullifying their immediate strength in number.[24] However, the Middle Gate had one major

[19] Herodotus, *The Histories*, 7.205.
[20] Herodotus, *The Histories*, 7.202.
[21] Herodotus, *The Histories*, 7.60.
[22] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 55.
[23] Tom Holland. *Persian Fire. The First World Empire and the Battle for the West*. Anchor Books, 2005, p. 222.
[24] Holland, *Persian Fire*, p. 212.

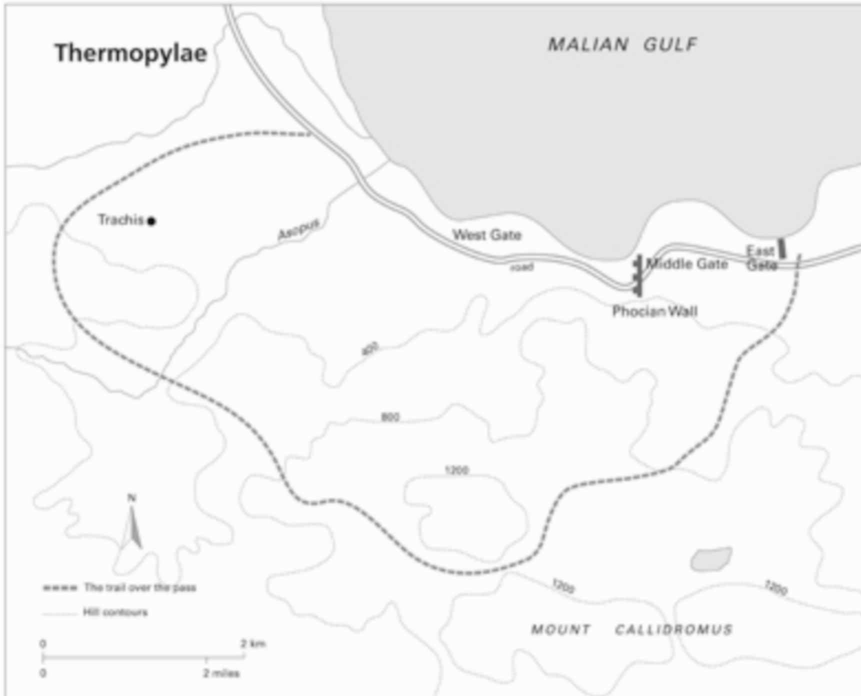


Figure 4: The "Middle Gate" and hidden path through the mountains. Holland, *Persian Fire,* p. 222.

weakness: a hidden path through the mountains that met the main road behind the Greek position. When he had been informed about this flaw by locals, Leonidas sent a contingent of 1,000 Phocians from his army to guard the path.[25]

When the Persian army finally arrived, Xerxes sent a scout to investigate the Greeks. When the scout arrived at the Greeks' position, he saw only a few Spartan men bathing and relaxing, as was customary before battle. He informed Xerxes of the state of the Greek army and the unnatural sight he had encountered. However, one of Xerxes's trusted advisors warned the king that the Spartan hoplites should not be taken at face value, as they were widely regarded as the greatest soldiers in all of Greece.[26] Nevertheless, Xerxes remained unswayed by this news and believed that the Greek army would surely retreat at the sight of his much larger force. After four days of waiting, however, to Xerxes's dismay, the Greeks showed no signs of leaving. So, on the morning of the fifth day, Xerxes began his assault.[27]

The early Persian waves were composed of Medes who had been new additions to the Persian army. Trained to fight in rough terrain using their overwhelming numbers to overpower the enemy, they were unprepared to fight the Greeks in the close quarters of the Middle Gate's bottleneck.[28] Herodotus explains,"The Medes fell upon the Greeks in a great rush, and although many were slain, others still pressed

[25]Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 62.
[26]Herodotus, *The Histories*, 7.209.
[27]Herodotus, *The Histories*, 7.210.
[28]Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 45.

the attack, and were not driven back, despite terrible losses."[29] In the early stages of the battle, to their surprise, the Persians were unable to penetrate the Greeks' defensive position. As Leonidas had intended, the narrow path forced the Persians to break formation and fight in conditions for which they had not been trained.[30] The Greeks capitalized on this natural bottleneck by employing the phalanx formation to great effect. As a result, when the Persians arrived at the Greek position, they were met by an impenetrable wall of spears and shields. As fighting continued, the phalanx would push against the waves of Persians, inflicting heavy damage. In some cases, the Spartan contingent of the Greek army would fein a retreat, inciting the Persians to pursue them. When they did, the Spartans would turn, reforming their phalanx and inflicting massive casualties.[31,32]

At around midday, Herodotus recalls, "such was the mauling inflicted on the Medes that they were withdrawn from the attack, and replaced by the Persian 'Immortals'... and who, it was expected, would have no difficulty completing the task."[33] However, these more elite soldiers had similarly poor results, and were also withdrawn. On the second day, Xerxes continued to expect his army would easily defeat the Greeks. Surely, he believed, the small Greek army would show signs of fatigue from the first day and would be easily beaten. However, since the Greeks consistently substituted weary soldiers with fresh ones, they did not show signs of fatigue. Thus, the results of the second day were quite similar to the first. Infuriated by the results, Xerxes withdrew his forces early to consider alternative strategies.[34]

As he pondered how to defeat the resilient Greek army, Xerxes was approached by a local Greek farmer named Ephialtes.[35] Ephialtes advised the Persian king that he should send a portion of his army on a secret path through the mountains to flank the Greeks. Heeding the farmer's suggestion, Xerxes ordered his Immortals accordingly.[36] Early in the morning of the third day, Greek scouts noticed this maneuver and notified Leonidas. When the Greek regional leaders realized that they would be surrounded by the end of the day, many opted to retreat. The Spartans, who were taught to welcome death, remained with the Thebans and Thespians as a rear guard. Though they would surely die, those who stayed were determined to defend their allies' retreat.[37]

Aware that they were going to die, the Greeks defiantly advanced beyond their original position into a much wider section of the pass. Here, at "the time in the morning when market-squares fill up",[38] the

[29] Herodotus, *The Histories*, 7.210.
[30] Holland, *Persian Fire*, p. 212.
[31] Herodotus, *The Histories*, 7.211.
[32] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, pp. 65-66.
[33] Herodotus, *The Histories*, 7.211.
[34] Herodotus, *The Histories*, 7.212.
[35] Holland, *Persian Fire*, p. 220.
[36] Herodotus, *The Histories*, 7.213.
[37] Herodotus, *The Histories*, 7.222.
[38] Herodotus, *The Histories*, 7.223.

Greeks met the Persians in brutal combat. Herodotus describes:

> The Greeks, you see, because they knew that they were going to die at the hands of the men coming round the mountain, did all they could to demonstrate to the Barbarians the full force of their strength, and fought in a frenzy, without regard for their lives. Now, by this stage, because most of them had smashed their spears to splinters, they were using their swords to slaughter the Persians.[39]

As the fighting intensified, Leonidas was slain. Once they had recovered his body, the remaining Greeks withdrew to a small hill behind their original position. They were met on one side by the main Persian army and on the other, by the flanking Immortals. Here, the Greek held their final stand until each man was slain.[40]

3.2 Arms and Armor

The Greek army that fought at the Battle of Thermopylae was composed of citizen-soldiers from various regions of Greece. Since the average city-state, or *polis*, could not afford a standing army, wealthy citizens would buy their own weapons and armor and double as soldiers when necessary. However, as farmers and artisans, these citizen-soldiers lacked significant military training. As a result, they equipped "themselves as hoplites, heavily armored infantry who fought shoulder to shoulder in a large formation known as a phalanx."[41] This system was implemented by nearly every city-state in Greece, with one famous exception: the city-state known as *Lacedaemon*, or, more commonly, Sparta. Located in the southern plains of the Peloponnese, Sparta favored a militaristic community over the common agrarian system. Unlike other Greeks, Spartans began military training at a very young age. Over a period of roughly 14 years, these young boys were put through grueling tests of strength and endurance, which transformed them into the greatest fighting force in ancient history.[42] Herodotus praises the Spartans, saying, "The Lacedaemonians fought in a manner that richly merits description, and left no one in any doubt that, while they were men who really knew what they were about, their opponents were just amateurs."[43] Similarly armed as hoplites, the Spartans were clearly the only professional soldiers to fight at the Battle of Thermopylae.

Historically, hoplites wore Corinthian style helmets formed from a single sheet of either bronze or iron that protected the entire head, neck, and face (Figure 5). Inside the helmet, an inner lining made from leather added extra padding, dampening the effects of a blow to the head. The Corinthian helmet, which

[39] Herodotus, *The Histories*, 7.224.
[40] Herodotus, *The Histories*, 7.225.
[41] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 22.
[42] Bradford, *Thermopylae*, p. 46.
[43] Herodotus, *The Histories*, 7.211.

Figure 5: A Corinthian helmet on display at the Archaeological Museum of Olympia in Olympia, Greece. Photo by Josh Wasserman.

became a symbol of ancient warfare, gained recognition for its iconic shape and the plume of horse hair that decorated its crest.[44] The hoplite's torso was completely covered by a metal and leather corselet that extended to the top of the knee. Below the waist however, the corselet was cut into thin strips that afforded the same protection without limiting the soldier's movement. Lastly, metal greaves, specifically molded for the wearer, were strapped around the calf on both legs to protect the foot and shin.[45,46]

Perhaps the most significant piece of equipment the hoplite carried was the shield. On average, the shield measured three feet in diameter. Made with a sturdy wooden core covered and reinforced by hammered bronze, the round shield could deflect the strongest of attacks (Figure 6). As a result, the hoplite employed the shield with great effect in their dominant phalanx formation.[47] In the phalanx, hoplites stood shoulder to shoulder such that the shield carried in their left arm would cover the right side of the soldier at their side. Locking their shields together in this manner, formed an impenetrable wall of armor.[48] Often several lines deep, the formation easily maintained its integrity as soldiers could readily reinforce weak portions of the wall. Hoplites also carried six foot spears in their right hand that protruded through spaces in the shield wall. Designed for optimal balance and durability, these iron tipped spears could be thrust

[44] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 23.
[45] Bradford, *Thermopylae*, p. 57.
[46] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 24.
[47] Bradford, *Thermopylae*, p. 55.
[48] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 24.

Figure 6: A Spartan hoplite shield on display at the Museum of the Ancient Agora in Athens, Greece. Photo by Josh Wasserman.

with great accuracy and power, giving the phalanx a deadly range of nearly 8 feet.[49] If enemies managed to avoid the line of spears, however, hoplites could use their shields to shove them back. With their foe off-balance, hoplites equipped with light curved short swords would emerge from behind their shields and strike the enemy with deadly precision, only to return to safety behind their shields. This was the manner in which most hoplites fought.[50]

Unlike the Greek hoplite, the Persian foot soldier was equipped to fight on rough terrain and wore comparatively light armor. Their outfit consisted of minimal cloth or leather clothing and, occasionally, a metal helmet. They carried a light shield made of wicker and leather that was often shaped like a crescent or a figure eight.[51] The infantry was equipped with a variety of weapons including a short spear, a long double-edged dagger and a light battle axe.[52] Most importantly, however, the Persian infantry was equipped with a composite bow and a quiver of short arrows with bronze or iron heads. This bow was a focal point of the Persian strategy. In most situations, the Persian army would bombard the enemy with arrows, ideally thinning the ranks before engaging in hand to hand combat with their minimal body armor. However, at

[49] Bradford, *Thermopylae*, p. 56.
[50] Bradford, *Thermopylae*, p. 57.
[51] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 40.
[52] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 37.

the Battle of Thermopylae, the Greeks had positioned themselves in such a narrow section of the path that arrows would have had little effect. Thus, the Persians' only option was to charge directly into the Greek phalanx. With lighter armor and shorter spears, the Persians were ill-prepared to take the Greeks head on, as they were surely "easy prey for [the] wall of bronze-clad professional killers."[53]

In addition to the basic infantry, the Persians relied on their heavily trained and individually selected Immortals. The Immortals were known to be the best of the best in the Persian army, hand picked to serve in the king's guard. Justly called the "Immortals", this division of the Persian army was said to have contained 10,000 soldiers. With so many soldiers at their disposal it would have seemed that they had infinite reinforcements, or, in a sense, were immortal. However, it is evident that the Immortals merely had power in number since they similarly wore light cloth and leather armor and carried light weapons. Nevertheless their extensive training would have made them far superior to the typical foot soldier.[54]

4 The Simulation

The primary goal of this project was to produce an accurate model of the Battle of Thermopylae that could subsequently be modified to generate counter-factual results. In order to complete this task, I needed to develop a simulation that could test both the historical and counter-factual models. In designing this simulation, the only requirement was that it be robust enough to handle multiple different models without changing its core components. Over the course of the project, I approached this general requirement from various angles.

4.1 Java and Unity

My initial vision for this project was to develop a graphically based, computer game - type simulation that would show the interaction of individual autonomous agents replicating the battle. By using agents that possessed some common form of artificial intelligence, individual soldiers could behave independently of one another but also work well together as an army. In order to prove I could build a simulation of this complexity I first had to develop a simple interaction between just two of these agents. I designed this feature, dubbed "The Interaction" using the object-oriented programming language Java. With limited knowledge of software design, however, my final design, Figure 7, repurposed some project materials from CSC-260: Large-Scale Software Design. My previous work implemented a variety of different design strategies that incorporated class inheritance and encapsulation: two properties that would make classes

[53]Holland, *Persian Fire*, p. 212.
[54]Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 41.

more robust and reusable.

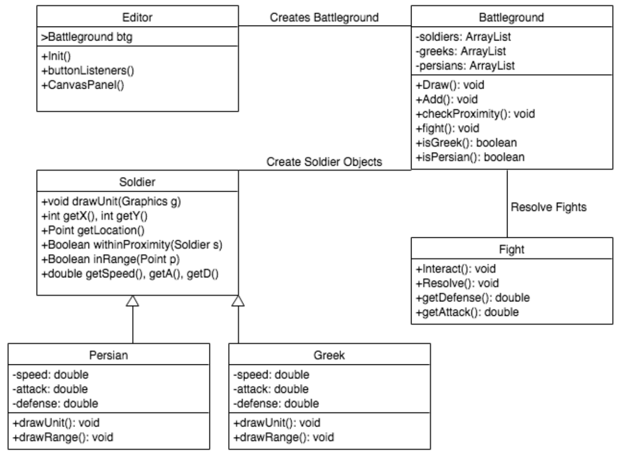


Figure 7: Class diagram of the simulation prototype.

The design for "The Interaction" had three main components, each individually responsible for a specific feature of the simulation to avoid having to share data across classes: *Editor*, *Battleground*, and *Soldier*. The front end of the program, the visual component, was managed exclusively by the *Editor* class, initializing the simulation and running the Java applet. At the start of the simulation, the *Editor* would create a new *Battleground* to store the simulation data and facilitate "The Interaction". The *Battleground* would creates the necessary *Soldier* objects to fill the simulation. The resulting prototype, Figure 8, produced text based results that indicated the outcome of a fight between two soldiers.

Figure 8: The Java applet as it appeared during prototyping.

My next step was to implement more complex features, namely movement and primary autonomous behaviors. However, after some trouble, I realized that Java did not have the correct tools to successfully and efficiently add this functionality into the simulation. I would have had to teach the program to treat soldiers as solid objects, sense and distinguish between hostile and friendly soldiers, give agents artificial intelligence that allowed them to operate individually and then ensure that each of these elements could run simultaneously. While Java is a very robust object-oriented programming language, implementing these components would have simply taken too much time.

I explored several game development tools that could potentially handle these increasingly complex demands before I discovered the *Unity* game engine. This open-source game development environment specializes in building cross-platform video games.[55] With physics, motion and sensory features included in the free version of the software, *Unity* provided all the necessary tools to pick up where my Java program left off. With help from *Unity's* massive online community, I was able to learn how to use this new environment with relative ease. But, since *Unity* is not an object-oriented programming language like Java, my original class-based design was no longer applicable. However, my original design, Figure 7, did divide the program's logic into distinct chunks that I could use to make the transition slightly easier. Fortunately, many of the features in my Java prototype came as standard features of the *Unity* engine. This meant that I did not have to reimplement any of the visual components in the EDITOR class. By the end of the first term, I successfully implemented motion and sensory elements into "The Interaction", Figure 9, leaving the implementation of artificial intelligence and the historical model to the next development phase. Moving into the second half of the project, however, it became evident that *Unity* would also struggle to scale easily given the demanding nature of my simulation.

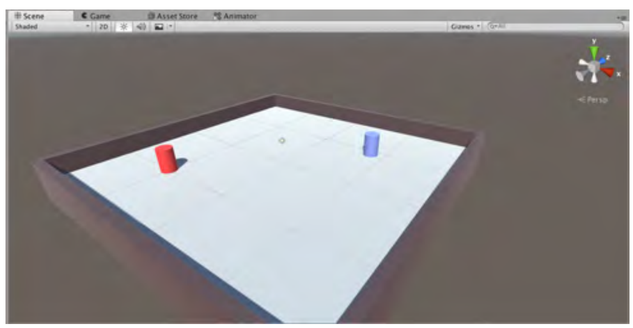


Figure 9: "The Interaction" as it appeared in *Unity*.

4.2 SimPy

In the end, I built my simulation in Python using the module SimPy, "a process-based discrete-event simulation framework."[56] The SimPy framework splits a simulation's primary components into three distinct functions: ENVIRONMENT, PROCESSES, and EVENTS. The ENVIRONMENT tracks the simulation time and data, and stores its list of PROCESSES and EVENTS.[57] These PROCESSES are used to define the model's behavior and execute the simulation. They do this by interacting with the ENVIRONMENT and other PROCESSES

[55] *Unity Documentation*. 5.5. Unity Technologies. Oct. 2005, p. 2.
[56] *SimPy Documentation*. 3.0.10. Team SimPy. Jan. 17, 2017, p. 1.
[57] *SimPy Documentation*, p. 10.

using the EVENTS.[58] By dividing key components into these three functions, the SimPy module provides Python users with an easy and dynamic tool with which to build simulations with precisely scheduled events and customizable model behaviors.

Choosing to develop the simulation using SimPy did, however, demand that I change my original design. In SimPy, my simulation would no longer focus on a visual representation of the battle. Instead, the primary goal of my simulation was to generate quantitative results. This was an important realization. In the first half of the project I had focused only on developing a graphical simulation of the battle without considering how I would display any results. By switching to SimPy I realized that I had been too focused on the appearance of the simulation and had not even considered that quantitative results would need to be at its heart. Thus, the revised design for the simulation of the Battle of Thermopylae focused on producing quantitative results from a historical model defined according to the historical record.

4.3 Developing the Simulation Environment

In theory, the simulation ENVIRONMENT should be the only component of the simulation that changes with each counter-factual scenario. As a result, the historical model needs to be versatile and robust enough to apply to every possible scenario regardless of simulation length. It just so happens that each battle scenario accepts one important fact which I exploited to optimize the code's reusability: they each abide by the 24 hour day. Although this may seem like a trivial observation, it actually substantially simplifies event scheduling, a process which can easily lead to complex and redundant code. This is a direct result of the simulation ENVIRONMENT tracking time as a counter rather than a proper clock.

In order to address this, I divided a 24-hour day into three EVENTS: *morning*, the period from midnight to 7:00 a.m.; *daytime*, the period from 7:00 a.m. to 8:00 p.m. that contains all combat; *night*, the period from 8:00 p.m. to midnight (see Figure 10). To maintain the historical integrity of the simulation at this level, the duration of these events were assigned according to the average length of an autumn day on the Greek coast.[59] Thus, together, these periods accurately lasted 1,440 units of simulated time, or 24-hours in minutes. With the addition of a day counter, not only could I accurately schedule battle events to occur at specific times, but I also could schedule them to specific days. This means that the simulation could run correctly regardless of the simulation length or scenario. Without this design, for example, in order to schedule the *lunch* event to occur precisely at noon each day, I would need to tell the program to process this event at simulation times 720, 2160, 3600, etc. Eventually, when the simulation length is changed, hard coding the schedule in this manner would become an issue since *lunch* would not be scheduled for the additional days.

[58] *SimPy Documentation*, pp. 3-4.
[59] Steffen Thorsen. *Time and Date AS*. URL: `https://www.timeanddate.com/sun/greece/athens`.

However, with this design, I could easily schedule *lunch* to occur 300 minutes into the *daytime* event, which is noon on the 24-hour clock. Following this design made the program more robust and reusable by hiding the simulation length from the model.

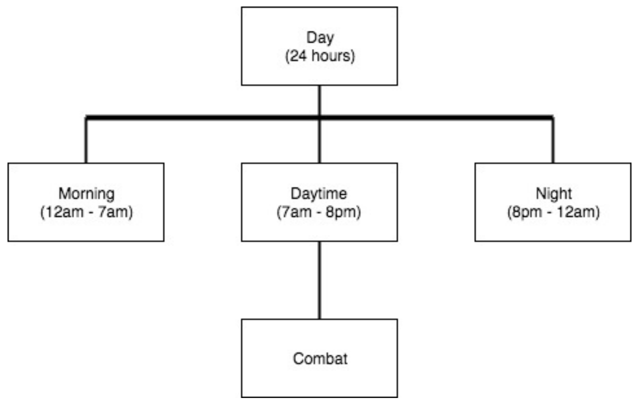


Figure 10: Hierarchical day structure.

4.4 Building The Model

With the core event hierarchy built into the simulation environment, I could begin development on the historical model. First, I created a timeline of key events and details from Herodotus's account. Since SimPy implements model behaviors using PROCESSES and EVENTS, I divided the elements of my timeline into two groups: *major* and *minor* events. Major events only occurred once and represented important moments of the battle. Minor events, on the other hand, were recurring and provided primary model behaviors. Thus, major events were implemented into EVENT type functions so they could be scheduled precisely, while the minor events were implemented as generic EVENT functions and PROCESSES. Since EVENTS employ PROCESSES to define their behavior, I added the *major* events to the model first.

The first *major* battle event is "Mountain Defense". Though this event technically took place prior to the beginning of the battle, it is an event that alters the population of the Greek army making it too important to omit. "Mountain Defense" simulates the Greeks' decision to have the Phocians defend the mountain path, removing them from the main army. The next *major* battle event, "Xerxes Withdraws", occurred on the second day and suspended fighting for the rest of the day as Xerxes did according to the historical record.[60] The final *major* battle event, "Ephialtes's Betrayal"[61], initiated the battle events "Greek Retreat"[62] and "Final Stand". Together, these battle events established the final stand sequence which occurred on the third day

[60] Herodotus, *The Histories*, 7.212.
[61] Herodotus, *The Histories*, 7.213.
[62] Herodotus, *The Histories*, 7.222.

and resulted in the death of the remaining Greeks.[63] By implementing *major* events first, I ensured that they would not be able to directly alter the model's core behavior, making them non-essential to the simulation's success.

Like their counterparts, the *minor* events highlight recurring actions taken during the battle. These include: Spartan fein retreat tactics, changes in the Greek formation, and Persian assaults. Since variations of the minor events occurred several times throughout the battle, I implemented them as generic EVENT functions that change behavior according to simulation time. In some cases however, these recurring events happened at unspecified times and relied on implicit behaviors in order to function correctly. I implemented these implicit behaviors as PROCESSES. This way, unscheduled events could be triggered once their conditional processes are completed. The execution of events and processes and the interaction between them affect the simulation by changing the value of specified variables in the simulation environment. In order to apply these changes to generate a quantitative result, the model was initialized with historical quantities from Herodotus's primary account and scholarly secondary sources.

4.5 Historical Quantities

Initial army totals were originally taken directly from Herodotus. Research revealed, however, that many scholars believe that Herodotus greatly exaggerated or miscalculated the size of the Persian army.[64] These sources suggest that the army was closer to 100,000 soldiers, not, as Herodotus claims, over 2,000,000.[65] Although this figure is too high, for the purposes of the historical simulation, it can be assumed that the Persian army was essentially infinite compared to the Greeks. Luckily, Herodotus cared far more about the Greeks and provides a detailed list of the number of soldiers in their army and where they come from:

> There were 300 hoplites from Sparta, and 1,000 altogether from Tegea and Mantinea, with 500 drawn from each, 120 from Arcadian Orchomenus, and 1,000 from the rest of Arcadia. In addition to the Arcadians, 400 came from Corinth, 200 from Phleious and 80 from Mycenae. These were the contingents from the Peloponnese, while from Boeotia there were 700 Thespians and 400 Thebans. Swelling these numbers were the Locrians of Opous - who, when the call went out, sent their entire fighting force - and 1,000 Phocians.[66]

He also recalls, in significant detail, the effect that Epialtes' betrayal had on the Greeks. Herodotus states that after some Greeks decided to retreat, "there remained with the Lacedaemonians only the Thespians

[63] Herodotus, *The Histories*, 7.223 - 226.
[64] Fields and Noon, *Thermopylae 480 BC: Last Stand of the 300*, p. 55.
[65] Herodotus, *The Histories*, 7.184.
[66] Herodotus, *The Histories*, 7.202-203.

and the Thebans."[67] This information is easily translated for use in the simulation.

At this point, however, Herodotus is of little use. His account scarcely describes actual fighting and, when it does, it lacks any real substance, as he tends to substitute empirical data with more flowery language. As a result, a significant amount of research went into calculating the outcome of each violent interaction. In the simulation, as in the battle, each period of combat lasts until the wave of Persian soldiers was either completely destroyed or forced to retreat. Until this point, fighting would have been continuous and deadly. Herodotus states, "The Medes fell upon the Greeks in a great rush, and although many were slain, others still pressed the attack, and were not driven back, despite terrible losses."[68] Although he fails to provide exact data, Herodotus does succeed in informing the reader about the pace of battle. He also reports the results of these bloody conflicts by saying, "... there was no counting the number who fell."[69] Although Herodotus does not provide concrete information about the number of soldiers who died in these epic fights, he does establish a relatively high expectation for the number of soldier who died during these epic fights.

At the end of his account of the Battle of Thermopylae, Herodotus estimates that roughly 20,000 Persians[70] and 4,000 Greeks[71] were killed over the three days. Importantly, however, Herodotus mentions that this estimated figure of dead Greeks consisted of"exclusively Lacedaemonians and Thespians" but also a group of helots. Since this is the first Herodotus mentions helots, Spartan slaves, my simulation does not take them into account. Additionally, other sources suggest that the total number of dead, including the roughly 900 helots, was closer to 2,300.[72]

4.6 Combat System

Unquestionably the most important process in the entire simulation was the combat system. This component is solely responsible for simulating the results of battle, and without it, the simulation would not produce any data. In order to properly develop this combat system, I had to understand every aspect of the combat between the Persians and the Greeks. In accordance with the historical record, I scheduled combat to occur during the daytime. Since the duration of combat was dependent on the population of the Persian wave, I implemented the combat sequence so that it was not dependent on a strict schedule. As a result, I programmed the combat sequence to run in one minute increments until the entire wave was either destroyed or routed. Each simulated minute of combat would first accumulate relevant data on the current

[67] Herodotus, *The Histories*, 7.221.
[68] Herodotus, *The Histories*, 7.210.
[69] Herodotus, *The Histories*, 7.211.
[70] Herodotus, *The Histories*, 8.24.
[71] Herodotus, *The Histories*, 8.25.
[72] Bradford, *Thermopylae*, p. 115.

state of the battlefield: specifically, the remaining wave population and the number of Persians and Greeks engaged during that minute of combat. This information is then sent to a *fight* PROCESS that resolves interactions between individual Persian soldiers and the Greeks. The *fight* process specifically iterates through the population of engaged Persian soldiers because combat is conditionally determined by the state of the Persian wave. Additionally, the result of a Persian's interaction is necessarily binary, either the Persian or Greek is killed; the reverse would need to account for the chance that a Greek does not fight or fights more than once, resulting in more potential outcomes.

In order to resolve these individual interactions, I developed a combat system that accounted for the variations in soldier quality and the relative randomness of battle. I implemented these factors using a separate *Soldier* class. Each soldier object represented the entire population of an individual type of soldier. At the Battle of Thermopylae, although there were many soldiers from various regions of the Mediterranean world, I could narrow these populations down to just four distinct types of soldier. For the Greeks, Spartans and citizen-soldiers, and for the Persians, Immortals and miscellaneous infantry. Each object required three parameters which distinguished them from other soldiers: name, army and population. I used the "name" parameter to easily reference soldier objects by name, "army" to indicate whether the soldier was Greek or Persian, and "total population" to indicate the number of soldiers remaining. These soldier objects were created at the beginning of the simulation and were assigned initial populations in accordance with their historical quantities. Soldier objects carried a number of important variables that were consistently updated to reflect the object's current state or fighting ability. This fighting ability was split into attack and defense skills and a number of state-dependent modifiers.

I determined a soldier's attack and defense values based on the quality of their equipment and their military skill relative to the other soldier types. For simplicity, I graded these traits separately on a 5 point scale, then added the grades together to determine their value (Table 1) . To assess the quality of arms, I based my grade on the information stated in the Arms and Armor section of this paper. These sources allowed me to conclude that the arms and armor used by hoplites was consistently superior to that of the Persians. In fact, Ernle Bradford, an expert on the ancient mediterranean world, confidently concludes that, "Although [the Persians were] admirably equipped and trained for the type of warfare that they normally encountered – for instance, in their recent action against Egypt – they were not a match for the heavily armored Greek hoplite."[73] Additionally, as stated before, their arrows were likely too weak to penetrate the hoplite armor and, in the bottleneck, too inaccurate to hit the Greeks. As a result, I gave hoplites the highest rating, 5, and the standard Persian infantry the lowest, 1. Since Immortals wore armor slightly more protective than the main infantry, they received an armor rating of 2.

[73]Bradford, *Thermopylae*, p. 58.

The military skill attribute was ranked in a similar manner. Since the Spartans are still considered amongst the greatest warriors in history, they were easily assigned the highest skill rating of 5. Setting this standard, I easily assigned the Persian infantry the lowest possible rating. This decision was based on two facts. First, as Bradford mentioned, the infantry training did not prepare them for fighting in level, confined spaces. Secondly, having such a large army, the Persians prioritized quantity of soldiers over quality, training their infantry to fight in large groups rather than individually. Since they were provided some training I gave them an attack skill of 1, but, since they relied on number rather than skill I gave them a 0 defense skill. I graded the Immortals using similar logic. These soldiers would have been quite skilled as they were hand selected to serve in the kings guard. Like the infantry, however, Immortals used their large numbers to their advantage. In fact, Immortals were so famous for this quantity-focused style of fighting that their title, 'Immortal', stemmed from the illusion that when a soldier was killed another was there to take his place.[74] Therefore I gave them an attack skill of 2 but a defense skill of 0, for they too would have been ill prepared to fight at Thermopylae. Lastly, the citizen-soldiers that composed the bulk of the Greek army were required to pay for their own military training. This training prepared these citizen-soldiers to fight as hoplites. As a result, a large portion of their training taught them to rely on their excellent equipment. Consequently, I afforded them a skill rank of 2 both in attack and defense.

Soldier	Attack	Defense
Spartan	9	9
Citizen-Soldier	7	7
Immortal	4	2
Persian Infantry	2	1

Table 1: Soldier attack and defense attribute values

Depending on the state of the battle or the condition of the soldier, however, a modifier is applied to the attack and defense values. These modifiers account for factors which improve or detract from the soldier's fighting ability (Table 2). In accordance with the historical record, I created four modifiers for the Greeks and only one for the Persians. When the Greeks are in a phalanx, their attack and defense is improved by one and two points respectively. In the event that the formation is broken, the Greeks receive a *Frenzy* modification that improves their attack by two points but reduces their defense by one. Herodotus mentions that the Greeks fought even when they had destroyed their weapons, as a result I created a *Broken Weapons* modifier that reduced their attack and defense by five. Lastly, when the Spartans use their fein retreat tactic their attack is improved by two. The only modifier I created for the Persians improved their attack by two when they were not forced to fight in the bottleneck, benefiting from their strength in number.

The second component of this system was the combat "roll". Like the roll of the dice, this roll randomly

[74] Herodotus, *The Histories*, 7.83.

Modifier	Attack	Defense	Soldier Type
Phalanx	+1	+2	Greek
Frenzy	+2	+1	Greek
Broken Weapons	-5	-5	Greek
Fake Retreat	+2	N/A	Spartan
Strength in Number	+2	N/A	Persian

Table 2: Soldier attribute modifiers

selects a value from a range of integers with equal probability. However, unlike the typical six-sided die, the range of possible numbers generated by the combat die was equal to the attack or defense value. For example, if a soldier had an attack value of 5, the *attack roll* would produce a random number between 0 and 5. In the simulation, a *fight* compares the result of one soldier's *attack roll* with the result of the opponent's *defense roll*. The soldier with the higher roll value wins the fight, killing his foe. In the event of a tie, I decided that the defender would win the fight. In most cases, however, since the Greeks primarily fought in a phalanx, one roll was insufficient. Thus, when the Greeks were in formation, the fight would have two phases. In the first phase, the Persian soldiers compared their defense roll to the Greek's attack roll, illustrating the phalanx's deadly wall of spears. If the Persian has a lower roll, he is killed and the simulation iterates to the next Persian. However, if the Persian manages to produce a higher roll, the Greek is not killed and the Persian moves into the second phase. In this second phase, the Persian produces an attack roll and the Greek produces a defense roll. Unlike in the previous stage, the soldier with the lower roll dies.

Together, these two components formed the combat system that generated the results of combat. In the model, this system was implemented as the primary PROCESS of the combat sequence triggered in the *Daytime* EVENT. At the start of the *daytime* event, the simulation initializes a combat sequence, creating a new Persian wave and choosing which division of the Greek army is active. Next, the simulation enters a loop that continues to trigger the *fight* process until either the Persian wave or Greek division is destroyed. Each iteration of the loop lasts one simulated minute such that the *fight* process resolves one minute of combat at a time. As described above, *fight* uses a system of rolls to resolve each Persian-Greek interaction individually. Once the *process* has produced a result for each interaction, it returns the results to the simulation environment. Here, the simulation data is updated and the next simulated minute begins. The simulation returns the final results of the battle once the entire population of Greeks or Persians is killed or the assigned simulation time is reached.

5 Results

Before I could start simulating counter-factual simulations I had to ensure that my historical model produced historically accurate data. Thus, the results of the historical simulations needed to fulfill two important criteria. First, the model had to faithfully imitate the course of the battle. Since the simulation ends when the entire population of Greeks or Persians is killed, the simulation of the historical model had to end during the Greeks' final stand on the third day. Although I scheduled each of the battle's EVENTS to occur precisely in accordance with the historical record, the final stand sequence was scheduled to begin on the third day and continue until the end of the simulation. In theory, this meant that it was possible for the final stand sequence to continue into the fourth or fifth day. The second criterion required that the simulation result in historically accurate death totals. As previously mentioned, however, the death toll is a topic of debate, with the estimates of Greek deaths ranging from 2,000 to 4,000. Since my historical model does not account for the populations of helots or the 1,000 Phocians after they are sent to guard the mountain pass, the target result for Greeks killed is in the range of 1,500 to 2,000.

		Day 1	Day 2	Day 3	Total
Greek	Average	168.48	42.06	1345.36	1555.9
	Standard Deviation	14.32	8.34	7.38	14.48
Persian	Average	9811.72	3804.34	7253.4	20869.46
	Standard Deviation	125.06	70.88	18.89	130.32

Table 3: Results of historical simulations.

I ran the historical simulation a total of 50 times (Table 3). With this many trials, I could ensure that the simulation results were consistent and that the potential margin of error and standard deviation was low relative to the army population. Over these 50 simulations, the battle consistently concluded on the third day during the final stand sequence. Furthermore, the average simulated death totals for both the Greeks and the Persians fell within the estimated range. On average, these simulations resulted in the death of 1,555.9 Greeks and 20,869.46 Persians. Perhaps more telling of the model's accuracy, however, is the standard deviations in the data it produced. Results deviated from the mean death total by an average of only 14.48 Greeks and 130.32 Persians. To put these values into perspective, the standard deviation from the average total of Greek deaths represents only .2% of the total population and only .9% of the average death total. For the Persians, these deviations are representative of only .1% of the total population and .6% of the average total deaths. In all cases, the model produced standard deviation that represented less than

1% of the target population. These results show that my model consistently produces accurate historical data. Thus, I can use the model to accurately simulate "What-If" scenarios with confidence.

5.1 What if Ephialtes never betrayed the Greeks?

At the end of the second day, the Battle of Thermopylae appeared to have no obvious conclusion. However, Ephialtes's betrayal of the Greeks led to their untimely demise on the following day. This begs the question – What if Ephialtes never betrayed the Greeks? Although we cannot say for sure, based on the course of the battle to that point, it would seem that the battle would have lasted several more days. By removing Ephialtes's betrayal from the historical model, the simulation can reveal what the outcome of the battle might have looked like.

In order to simulate this counter-factual scenario, I had to modify some aspects of the historical model. First, I removed the *Ephialtes's betrayal* sequence from the simulation schedule. This meant that the *Greek Retreat* and *Final Stand* events never occur, effectively allowing the simulation to run for more than three days. Next, I added a random strategy component for the Persians. Since, we cannot know how Xerxes would have strategized the following days, this random strategy generator randomly selected which division of the Persian army was fighting in a combat sequence. Finally, I added fatigue as a factor. In the historical model, fatigue only became a factor during the final stand, in accordance with Herodotus's record, however, since the battle would presumably last longer than three days, fatigue would become a serious factor starting on the third day.

		Day 1	Day 2	Day 3	Total
Greek	Average	170.34	41.68	178.42	390.44
	Standard Deviation	13.88	6.72	17.69	23.15
Persian	Average	9823.26	3820.7	9443.92	23087.88
	Standard Deviation	126.76	75.47	433.03	446.57

Table 4: Results of the counter-factual simulation that removed *Ephialtes's Betrayal* as a factor

I ran the modified model through the simulation 50 times (Table 4). Without Ephialtes's betrayal as a factor, the battle continued for an average of 17.6 days. Additionally, in each of these 50 simulations, the Greeks managed to outlast and defeat the Persian army. Compared to the historical simulation results, the impact of Ephialtes's betrayal is obvious. Not only did the battle continue for nearly 18 days, but also the number of Greeks that died in the first three days is exactly 75% fewer than in the historical simulation. Additionally, the absence of a final stand on the third day results in 87% fewer Greek deaths. The results for the Persians remain consistent with the historical simulation during the first two days. On the third day, however, the standard deviation from the mean total is nearly three and a half times higher than in the historical simulation.

5.2 What if the battle was not fought during the Carnea Festival?

Next, I considered what impact the timing of the battle had on the outcome. In this scenario, I envisioned that the battle did not coincide with the Spartan Festival of Carnea which withheld the majority of the Spartan army from fighting in the Battle of Thermopylae. Compared to the previous counter-factual scenario, this one demanded very few modifications. Starting from the original historical model, I changed the population of Spartans in the Greek army to 10,000 and removed all the other Greek soldiers and, subsequently, the *Greek Retreat* caused by Ephialtes's betrayal. Without making any other modifications to the historical model, the simulation would illustrate the effect of having the full power of the Spartans fighting at the Battle of Thermopylae.

		Day 1	Day 2	Day 3	Total
Greek	Average	111.32	27.62	563.22	702.16
	Standard Deviation	12.13	4.69	76.19	79.61
Persian	Average	9764.68	3738.78	42741.36	56244.82
	Standard Deviation	115.11	69.17	450.61	458.46

Table 5: Results of the counter-factual simulation that removed the *Festival of Carnea* as a factor.

After running the simulation 50 times, the impact of the Spartan army was consistently obvious (Table 5). Although the Greeks were defeated in each of the 50 trials, the Spartans always survived into the second day of their final stand. As a result, the battle consistently lasted four days. In the first three days of the battle, the Spartans consistently outperformed the Greek army of the historical model. In fact, fewer Spartans died in the first three days of battle than the Greeks on the third day alone. Additionally, the Spartans killed over twice as many Persians on the third day as the Greeks did in the entire historical simulation, resulting in the death of roughly 56,244 Persians.

5.3 What if the battle was not fought at the "Middle Gate"?

The final "What-if" scenario that I simulated considered how the location impacted the outcome of the battle. Leonidas deliberately chose to position his army at a natural bottleneck that was, at most, 15 meters wide. This gave the Greeks a significant defensive advantage that negated the Persians' strength in numbers. However, if the battle had not occurred at Thermopylae or Leonidas chose to defend a different position on the path, the Greeks would not have benefitted from the advantages of the narrow bottleneck. In order to model this scenario, I had to make four important modifications. First, I widened the Greeks' position from 15 meters to 50 meters, which also increased the number of soldiers in a phalanx line from 20 to 55. Since, presumably, this would also suggest a change in location, the secret path through the mountains would not be a factor. Thus I removed the *Mountain Defense*, *Ephialtes's Betrayal* and *Final Stand* events from the simulation. Additionally, fighting in a larger space, the Persian army would not be forced to fight with fewer soldiers. As a result, I gave the Persians the *Strength in Number* modifier for the entire battle. Finally, since more than double the number of Greeks would be needed to form the phalanx, soldiers would not have enough time to rest in between waves and substitutions. Thus, I added fatigue as a factor for the duration of the battle.

		Day 1	Day 2	Day 3	Total
Greek	Average	313.74	303.4	172.32	789.46
	Standard Deviation	36.06	38.72	41.28	82.21
Persian	Average	12469.58	12436.46	7979.66	32885.7
	Standard Deviation	138.99	284.79	949.88	1014.21

Table 6: Results of the counter-factual simulation that removes the natural 15 meter-wide bottleneck as a factor.

In each of the 50 trials, the Greek army defeated the Persians in an average of 14.86 days (Table 6). Though they were able to survive longer outside of the bottleneck, the Greek army experienced nearly twice as many casualties on the first day. Compared to the results of the first counter-factual scenario, outside the bottleneck, the Greeks concluded the battle three days faster but, similarly, with twice as many casualties in the first three days. This is most likely the direct result of increasing the width of the battlefield. With roughly 110 Persians engaged in combat every minute, compared to 40 in the historical model, the chance that a Greek was killed was increased by 275%. Additionally, the *Strength in Number* modifier improved the odds of a Persian victory by 2 points on the offensive. From this perspective, the number of Greek casualties is actually relatively low.

5.4 Conclusions

The results of the counter-factual simulations explain the significance of the specific circumstances of the battle to its historical outcome. When we eliminate Ephialtes's betrayal and the Greek army's defensive position at the "Middle Gate" in these simulations, the Greeks triumphed over the Persians. Similarly, when the timing of the battle does not coincide with the Festival of Carnea, although the Greeks are still defeated, the superior Spartan army led to drastically increased Persian fatalities. In the absence of Ephialtes's betrayal, the Festival of Carnea, and the Greeks' defensive position at the "Middle Gate", the Greek army consistently out performed the historical scenario. Thus, we can conclude that each of these factors negatively impacted the Greek army's longevity, contributing to their ultimate defeat at the Battle of Thermopylae.

6 Future Work

Although the results of these counter-factual simulations adequately answered my questions, they also raised a number of questions that highlight some oversights in the simulation's implementation. For starters, I programmed the simulation to continue until either the Greeks or Persians were completely wiped out. Realistically, however, Xerxes and the Persian army would have retreated long before it got to this point. Although the absence of this variable does not negatively effect the simulation results, it does unrealistically indicate the length of these counter-factual battles. The Persians also had a limited amount of supplies: a factor which would similarly effect their ability to continue fighting after a certain time. In order to implement these variables into my model, I would have to establish some metric with which I could determine the effect that limited supplies and deaths would have on the state of the Persian army and Xerxes's willingness to keep fighting. Furthermore, the second counter-factual scenario highlights the important role played by the Festival of Carnea. Although the results revealed the impact that the battle's timing had on the outcome, it also raises the question– What if the Greeks resisted the Persians long enough to receive reinforcements? Historically, this was the Spartan's intended plan. Therefore, this variable would play a significant role in the other two counter-factual scenarios, which both lasted beyond the festival's conclusion.

If I were to continue the project in the future, I would primarily focus on the implementation of these important variables. With them, the model would produce counter-factual results that had more realistic depth, allowing the simulation to consider the significance of much smaller details of the battle. Secondly, I would overhaul certain aspects of the current model that generated inconsistent results: specifically, the random strategy generator. Eliminating these inconsistencies would reduce the standard deviation of simulation results, producing more precise data. These improvements would raise my confidence in the accuracy of the models results, allowing me to make more significant conclusions about the battle.

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